CHAPTER 4

CIRCUIT THEOREMS

Our schools had better get on with what is their overwhelmingly most important task: teaching their charges to express themselves clearly and with precision in both speech and writing; in other words, leading them toward mastery of their own language. Failing that, all their instruction in mathematics and science is a waste of time.

—Joseph Weizenbaum, M.I.T.

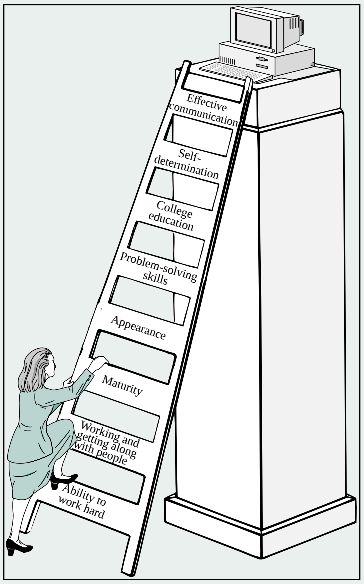
Enhancing Your Career

Enhancing Your Communication Skill Taking a course in circuit analysis is one step in preparing yourself for a career in electrical engineering. Enhancing your communication skill while in school should also be part of that preparation, as a large part of your time will be spent communicating.

People in industry have complained again and again that graduating engineers are ill-prepared in written and oral communication. An engineer who communicates effectively becomes a valuable asset.

You can probably speak or write easily and quickly. But how *effectively* do you communicate? The art of effective communication is of the utmost importance to your success as an engineer.

For engineers in industry, communication is key to promotability. Consider the result of a survey of U.S. corporations that asked what factors influence managerial promotion. The survey includes a listing of 22 personal qualities and their importance in advancement. You may be surprised to note that "technical skill based on experience" placed fourth from the bottom. Attributes such as self-confidence, ambition, flexibility, maturity, ability to make sound decisions, getting things done with and through people, and capacity for hard work all ranked higher. At the top of the list was "ability to communicate." The higher your professional career progresses, the more you will need to communicate. Therefore, you should regard effective communication as an important tool in your engineering tool chest.



Ability to communicate effectively is regarded by many as the most important step to an executive promotion.
(Adapted from J. Sherlock, *A Guide to Technical Communication.* Boston, MA: Allyn and Bacon, 1985, p. 7.)

Learning to communicate effectively is a lifelong task you should always work toward. The best time to begin is while still in school. Continually look for opportunities to develop and strengthen your reading, writing, listening, and speaking skills. You can do this through classroom presentations, team projects, active participation in student organizations, and enrollment in communication courses. The risks are less now than later in the workplace.

4.1 INTRODUCTION

A major advantage of analyzing circuits using Kirchhoff's laws as we did in Chapter 3 is that we can analyze a circuit without tampering with its original configuration. A major disadvantage of this approach is that, for a large, complex circuit, tedious computation is involved.

The growth in areas of application of electric circuits has led to an evolution from simple to complex circuits. To handle the complexity, engineers over the years have developed some theorems to simplify circuit analysis. Such theorems include Thevenin's and Norton's theorems. Since these theorems are applicable to *linear* circuits, we first discuss the concept of circuit linearity. In addition to circuit theorems, we discuss the concepts of superposition, source transformation, and maximum power transfer in this chapter. The concepts we develop are applied in the last section to source modeling and resistance measurement.

4.2 LINEARITY PROPERTY

Linearity is the property of an element describing a linear relationship between cause and effect. Although the property applies to many circuit elements, we shall limit its applicability to resistors in this chapter. The property is a combination of both the homogeneity (scaling) property and the additivity property.

The homogeneity property requires that if the input (also called the *excitation*) is multiplied by a constant, then the output (also called the *response*) is multiplied by the same constant. For a resistor, for example, Ohm's law relates the input i to the output v,

$$v = iR \tag{4.1}$$

If the current is increased by a constant k, then the voltage increases correspondingly by k, that is,

$$\boxed{kiR = kv} \tag{4.2}$$

The additivity property requires that the response to a sum of inputs is the sum of the responses to each input applied separately. Using the voltage-current relationship of a resistor, if

$$v_1 = i_1 R \tag{4.3a}$$

and

$$v_2 = i_2 R \tag{4.3b}$$

then applying $(i_1 + i_2)$ gives

$$\boxed{v = (i_1 + i_2)R = i_1 R + i_2 R = v_1 + v_2} \tag{4.4}$$

We say that a resistor is a linear element because the voltage-current relationship satisfies both the homogeneity and the additivity properties.

In general, a circuit is linear if it is both additive and homogeneous. A linear circuit consists of only linear elements, linear dependent sources, and independent sources.

> A **linear circuit** is one whose output is linearly related (or directly proportional) to its input.

Throughout this book we consider only linear circuits. Note that since $p = i^2R = v^2/R$ (making it a quadratic function rather than a linear one), the relationship between power and voltage (or current) is nonlinear. Therefore, the theorems covered in this chapter are not applicable to power.

To understand the linearity principle, consider the linear circuit shown in Fig. 4.1. The linear circuit has no independent sources inside it. It is excited by a voltage source v_s, which serves as the input. The circuit is terminated by a load R. We may take the current i through R as the output. Suppose $v_s = 10$ V gives $i = 2$ A. According to the linearity principle, $v_s = 1$ V will give $i = 0.2$ A. By the same token, $i = 1$ mA must be due to $v_s = 5$ mV.

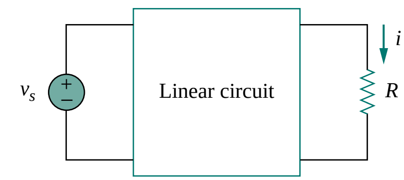


Figure 4.1 A linear circuit with input v_s and output i.

EXAMPLE 4.1

For the circuit in Fig. 4.2, find i_o when $v_s = 12$ V and $v_s = 24$ V.

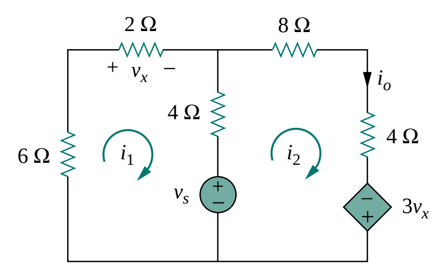


Figure 4.2 For Example 4.1.

Solution:

Applying KVL to the two loops, we obtain

$$12i_1 - 4i_2 + v_s = 0 \tag{4.1.1}$$

$$-4i_1 + 16i_2 - 3v_x - v_s = 0 \tag{4.1.2}$$

But $v_x = 2i_1$. Equation (4.1.2) becomes

$$-10i_1 + 16i_2 - v_s = 0 \tag{4.1.3}$$

Adding Eqs. (4.1.1) and (4.1.3) yields

$$2i_1 + 12i_2 = 0 \quad \Longrightarrow \quad i_1 = -6i_2$$

Substituting this in Eq. (4.1.1), we get

$$-76i_2 + v_s = 0 \quad \Longrightarrow \quad i_2 = \frac{v_s}{76}$$

When $v_s = 12$ V,

$$i_o = i_2 = \frac{12}{76} \text{ A}$$

When $v_s = 24$ V,

$$i_o = i_2 = \frac{24}{76} \text{ A}$$

showing that when the source value is doubled, i_o doubles.

PRACTICE PROBLEM 4.1

For the circuit in Fig. 4.3, find v_o when $i_s = 15$ and $i_s = 30$ A.

Answer: 10 V, 20 V.

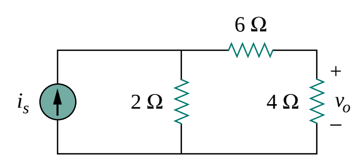


Figure 4.3 For Practice Prob. 4.1.

EXAMPLE 4.2

Assume $I_o = 1$ A and use linearity to find the actual value of I_o in the circuit in Fig. 4.4.

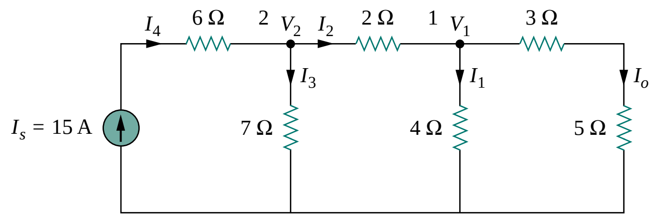


Figure 4.4 For Example 4.2.

Solution:

If $I_o = 1$ A, then $V_1 = (3+5)I_o = 8$ V and $I_1 = V_1/4 = 2$ A. Applying KCL at node 1 gives

$$I_2 = I_1 + I_o = 3 \text{ A}$$

$$V_2 = V_1 + 2I_2 = 8 + 6 = 14 \text{ V}, \qquad I_3 = \frac{V_2}{7} = 2 \text{ A}$$

Applying KCL at node 2 gives

$$I_4 = I_3 + I_2 = 5 \text{ A}$$

Therefore, $I_s = 5$ A. This shows that assuming $I_o = 1$ gives $I_s = 5$ A; the actual source current of 15 A will give $I_o = 3$ A as the actual value.

PRACTICE PROBLEM 4.2

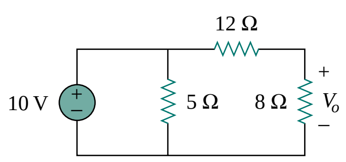


Figure 4.5 For Practice Prob. 4.2

Assume that $V_o = 1$ V and use linearity to calculate the actual value of V_o in the circuit of Fig. 4.5.

Answer: 4 V.

4.3 SUPERPOSITION

If a circuit has two or more independent sources, one way to determine the value of a specific variable (voltage or current) is to use nodal or mesh analysis as in Chapter 3. Another way is to determine the contribution of each independent source to the variable and then add them up. The latter approach is known as the *superposition*.

The idea of superposition rests on the linearity property.

> The **superposition** principle states that the voltage across (or current through) an element in a linear circuit is the algebraic sum of the voltages across (or currents through) that element due to each independent source acting alone.

Superposition is not limited to circuit analysis but is applicable in many fields where cause and effect bear a linear relationship to one another.

The principle of superposition helps us to analyze a linear circuit with more than one independent source by calculating the contribution of each independent source separately. However, to apply the superposition principle, we must keep two things in mind:

1. We consider one independent source at a time while all other independent sources are *turned off*. This implies that we replace every voltage source by 0 V (or a short circuit), and every current source by 0 A (or an open circuit). This way we obtain a simpler and more manageable circuit.
2. Dependent sources are left intact because they are controlled by circuit variables.

Other terms such as *killed*, *made inactive*, *deadened*, or *set equal to zero* are often used to convey the same idea.

With these in mind, we apply the superposition principle in three steps:

Steps to Apply Superposition Principle:

1. Turn off all independent sources except one source. Find the output (voltage or current) due to that active source using nodal or mesh analysis.
2. Repeat step 1 for each of the other independent sources.
3. Find the total contribution by adding algebraically all the contributions due to the independent sources.

Analyzing a circuit using superposition has one major disadvantage: it may very likely involve more work. If the circuit has three independent sources, we may have to analyze three simpler circuits each providing the contribution due to the respective individual source. However, superposition does help reduce a complex circuit to simpler circuits through replacement of voltage sources by short circuits and of current sources by open circuits.

Keep in mind that superposition is based on linearity. For this reason, it is not applicable to the effect on power due to each source, because the power absorbed by a resistor depends on the square of the voltage or current. If the power value is needed, the current through (or voltage across) the element must be calculated first using superposition.

For example, when current i_1 flows through resistor R, the power is $p_1 = Ri_1^2$, and when current i_2 flows through R, the power is $p_2 = Ri_2^2$. If current $i_1 + i_2$ flows through R, the power absorbed is $p_3 = R(i_1 + i_2)^2 = Ri_1^2 + Ri_2^2 + 2Ri_1i_2 \neq p_1 + p_2$. Thus, the power relation is nonlinear.

EXAMPLE 4.3

Use the superposition theorem to find v in the circuit in Fig. 4.6.

Solution:

Since there are two sources, let

$$v = v_1 + v_2$$

where v_1 and v_2 are the contributions due to the 6-V voltage source and

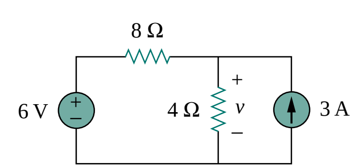


Figure 4.6 For Example 4.3.

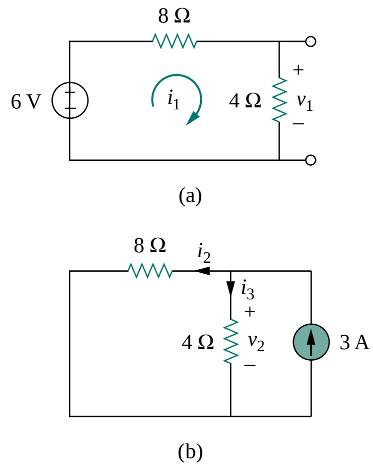


Figure 4.7 For Example 4.3: (a) calculating v_1, (b) calculating v_2.

the 3-A current source, respectively. To obtain v_1, we set the current source to zero, as shown in Fig. 4.7(a). Applying KVL to the loop in Fig. 4.7(a) gives

$$12i_1 - 6 = 0 \qquad \Longrightarrow \qquad i_1 = 0.5 \text{ A}$$

Thus,

$$v_1 = 4i_1 = 2 \text{ V}$$

We may also use voltage division to get v_1 by writing

$$v_1 = \frac{4}{4+8}(6) = 2 \text{ V}$$

To get v_2, we set the voltage source to zero, as in Fig. 4.7(b). Using current division,

$$i_3 = \frac{8}{4+8}(3) = 2 \text{ A}$$

Hence,

$$v_2 = 4i_3 = 8 \text{ V}$$

And we find

$$v = v_1 + v_2 = 2 + 8 = 10 \text{ V}$$

PRACTICE PROBLEM 4.3

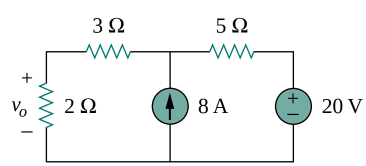


Figure 4.8 For Practice Prob. 4.3.

Using the superposition theorem, find v_o in the circuit in Fig. 4.8.

Answer: 12 V.

EXAMPLE 4.4

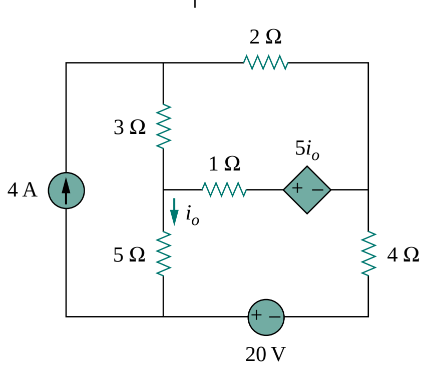


Figure 4.9 For Example 4.4.

Find i_o in the circuit in Fig. 4.9 using superposition.

Solution:

The circuit in Fig. 4.9 involves a dependent source, which must be left intact. We let

$$i_o = i_o' + i_o'' \tag{4.4.1}$$

where i_o' and i_o'' are due to the 4-A current source and 20-V voltage source respectively. To obtain i_o', we turn off the 20-V source so that we have the circuit in Fig. 4.10(a). We apply mesh analysis in order to obtain i_o'. For loop 1,

$$i_1 = 4 \text{ A} \tag{4.4.2}$$

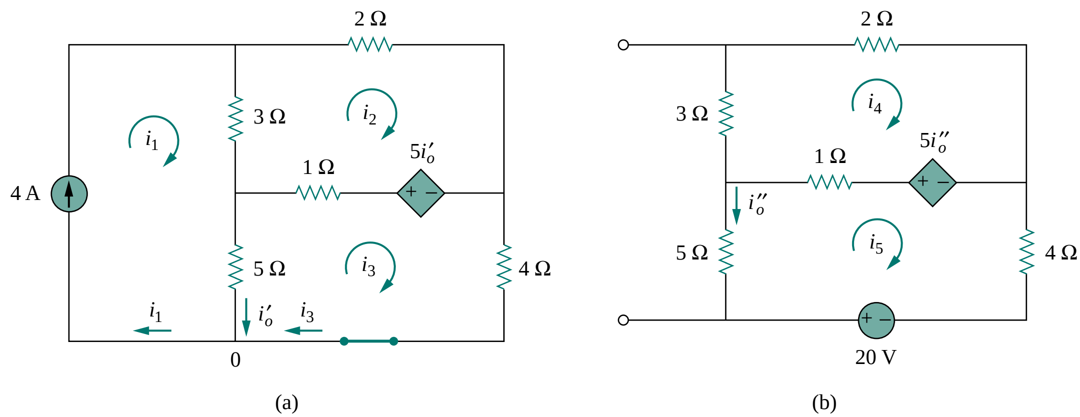


Figure 4.10 For Example 4.4: Applying superposition to (a) obtain i'_0, (b) obtain i''_0.

For loop 2,

$$-3i_1 + 6i_2 - 1i_3 - 5i'_o = 0 \tag{4.4.3}$$

For loop 3,

$$-5i_1 - 1i_2 + 10i_3 + 5i'_o = 0 \tag{4.4.4}$$

But at node 0,

$$i_3 = i_1 - i'_o = 4 - i'_o \tag{4.4.5}$$

Substituting Eqs. (4.4.2) and (4.4.5) into Eqs. (4.4.3) and (4.4.4) gives two simultaneous equations

$$3i_2 - 2i'_o = 8 \tag{4.4.6}$$

$$i_2 + 5i'_o = 20 \tag{4.4.7}$$

which can be solved to get

$$i'_o = \frac{52}{17}\text{ A} \tag{4.4.8}$$

To obtain i''_o, we turn off the 4-A current source so that the circuit becomes that shown in Fig. 4.10(b). For loop 4, KVL gives

$$6i_4 - i_5 - 5i''_o = 0 \tag{4.4.9}$$

and for loop 5,

$$-i_4 + 10i_5 - 20 + 5i''_o = 0 \tag{4.4.10}$$

But $i_5 = -i''_o$. Substituting this in Eqs. (4.4.9) and (4.4.10) gives

$$6i_4 - 4i''_o = 0 \tag{4.4.11}$$

$$i_4 + 5i''_o = -20 \tag{4.4.12}$$

which we solve to get

$$i_o'' = -\frac{60}{17}\text{ A} \tag{4.4.13}$$

Now substituting Eqs. (4.4.8) and (4.4.13) into Eq. (4.4.1) gives

$$i_o = -\frac{8}{17} = -0.4706\text{ A}$$

PRACTICE PROBLEM 4.4

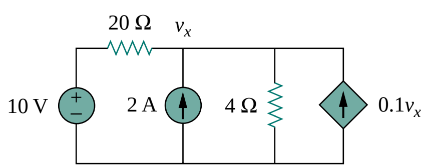


Figure 4.11 For Practice Prob. 4.4.

Use superposition to find v_x in the circuit in Fig. 4.11.

Answer: $v_x = 12.5$ V.

EXAMPLE 4.5

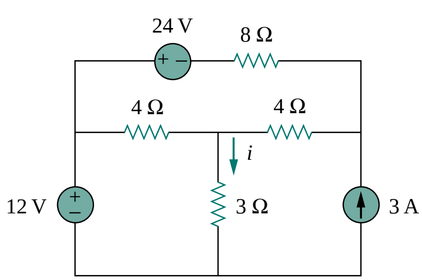


Figure 4.12 For Example 4.5.

For the circuit in Fig. 4.12, use the superposition theorem to find i.

Solution:

In this case, we have three sources. Let

$$i = i_1 + i_2 + i_3$$

where i_1, i_2, and i_3 are due to the 12-V, 24-V, and 3-A sources respectively. To get i_1, consider the circuit in Fig. 4.13(a). Combining 4 Ω (on the right-hand side) in series with 8 Ω gives 12 Ω. The 12 Ω in parallel with 4 Ω gives $12 \times 4/16 = 3$ Ω. Thus,

$$i_1 = \frac{12}{6} = 2\text{ A}$$

To get i_2, consider the circuit in Fig. 4.13(b). Applying mesh analysis,

$$16i_a - 4i_b + 24 = 0 \quad \Longrightarrow \quad 4i_a - i_b = -6 \tag{4.5.1}$$

$$7i_b - 4i_a = 0 \quad \Longrightarrow \quad i_a = \frac{7}{4}i_b \tag{4.5.2}$$

Substituting Eq. (4.5.2) into Eq. (4.5.1) gives

$$i_2 = i_b = -1$$

To get i_3, consider the circuit in Fig. 4.13(c). Using nodal analysis,

$$3 = \frac{v_2}{8} + \frac{v_2 - v_1}{4} \quad \Longrightarrow \quad 24 = 3v_2 - 2v_1 \tag{4.5.3}$$

$$\frac{v_2 - v_1}{4} = \frac{v_1}{4} + \frac{v_1}{3} \quad \Longrightarrow \quad v_2 = \frac{10}{3}v_1 \tag{4.5.4}$$

Substituting Eq. (4.5.4) into Eq. (4.5.3) leads to $v_1 = 3$ and

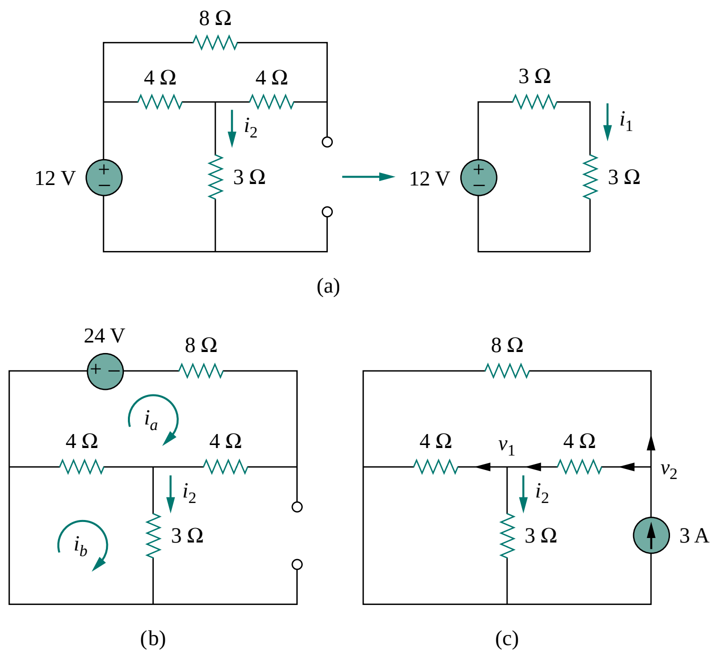


Figure 4.13 For Example 4.5.

$$i_3 = \frac{v_1}{3} = 1 \text{ A}$$

Thus,

$$i = i_1 + i_2 + i_3 = 2 - 1 + 1 = 2 \text{ A}$$

PRACTICE PROBLEM 4.5

Find i in the circuit in Fig. 4.14 using the superposition principle.

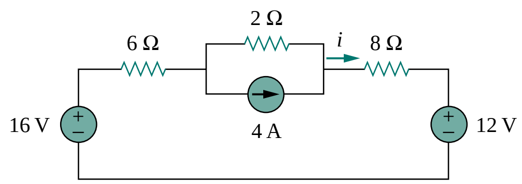


Figure 4.14 For Practice Prob. 4.5.

Answer: 0.75 A.

4.4 SOURCE TRANSFORMATION

We have noticed that series-parallel combination and wye-delta transformation help simplify circuits. *Source transformation* is another tool for simplifying circuits. Basic to these tools is the concept of *equivalence*.

We recall that an equivalent circuit is one whose v-i characteristics are identical with the original circuit.

In Section 3.6, we saw that node-voltage (or mesh-current) equations can be obtained by mere inspection of a circuit when the sources are all independent current (or all independent voltage) sources. It is therefore expedient in circuit analysis to be able to substitute a voltage source in series with a resistor for a current source in parallel with a resistor, or vice versa, as shown in Fig. 4.15. Either substitution is known as a *source transformation*.

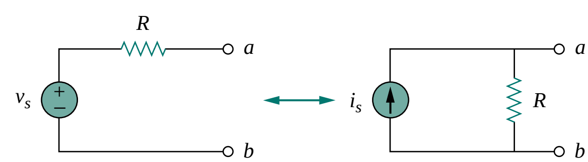


Figure 4.15 Transformation of independent sources.

A **source transformation** is the process of replacing a voltage source v_s in series with a resistor R by a current source i_s in parallel with a resistor R, or vice versa.

The two circuits in Fig. 4.15 are equivalent—provided they have the same voltage-current relation at terminals a-b. It is easy to show that they are indeed equivalent. If the sources are turned off, the equivalent resistance at terminals a-b in both circuits is R. Also, when terminals a-b are short-circuited, the short-circuit current flowing from a to b is $i_{sc} = v_s/R$ in the circuit on the left-hand side and $i_{sc} = i_s$ for the circuit on the right-hand side. Thus, $v_s/R = i_s$ in order for the two circuits to be equivalent. Hence, source transformation requires that

$$v_s = i_s R \qquad \text{or} \qquad i_s = \frac{v_s}{R} \tag{4.5}$$

Source transformation also applies to dependent sources, provided we carefully handle the dependent variable. As shown in Fig. 4.16, a dependent voltage source in series with a resistor can be transformed to a dependent current source in parallel with the resistor or vice versa.

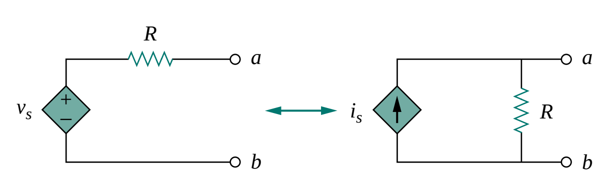


Figure 4.16 Transformation of dependent sources.

Like the wye-delta transformation we studied in Chapter 2, a source transformation does not affect the remaining part of the circuit. When

applicable, source transformation is a powerful tool that allows circuit manipulations to ease circuit analysis. However, we should keep the following points in mind when dealing with source transformation.

1. Note from Fig. 4.15 (or Fig. 4.16) that the arrow of the current source is directed toward the positive terminal of the voltage source.
2. Note from Eq. (4.5) that source transformation is not possible when $R = 0$, which is the case with an ideal voltage source. However, for a practical, nonideal voltage source, $R \neq 0$. Similarly, an ideal current source with $R = \infty$ cannot be replaced by a finite voltage source. More will be said on ideal and nonideal sources in Section 4.10.1.

EXAMPLE 4.6

Use source transformation to find v_o in the circuit in Fig. 4.17.

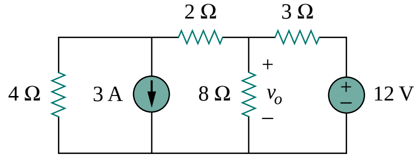


Figure 4.17 For Example 4.6.

Solution:

We first transform the current and voltage sources to obtain the circuit in Fig. 4.18(a). Combining the 4-Ω and 2-Ω resistors in series and transforming the 12-V voltage source gives us Fig. 4.18(b). We now combine the 3-Ω and 6-Ω resistors in parallel to get 2-Ω. We also combine the 2-A and 4-A current sources to get a 2-A source. Thus, by repeatedly applying source transformations, we obtain the circuit in Fig. 4.18(c).

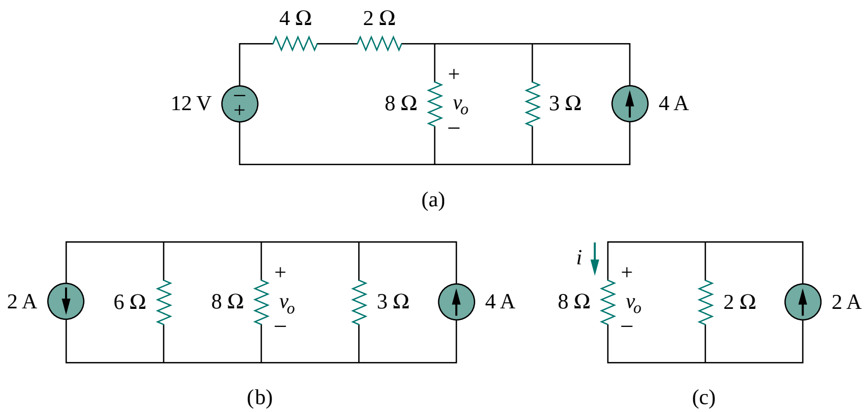


Figure 4.18 For Example 4.6.

We use current division in Fig. 4.18(c) to get

$$i = \frac{2}{2+8}(2) = 0.4$$

and

$$v_o = 8i = 8(0.4) = 3.2 \text{ V}$$

Alternatively, since the 8-Ω and 2-Ω resistors in Fig. 4.18(c) are in parallel, they have the same voltage v_o across them. Hence,

$$v_o = (8 \parallel 2)(2\text{ A}) = \frac{8 \times 2}{10}(2) = 3.2\text{ V}$$

PRACTICE PROBLEM 4.6

Find i_o in the circuit of Fig. 4.19 using source transformation.

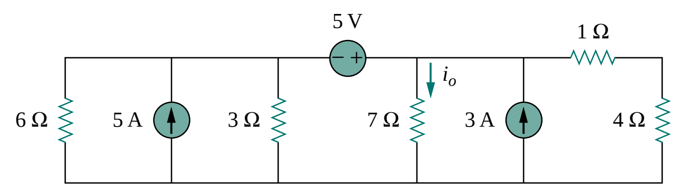


Figure 4.19 For Practice Prob. 4.6.

Answer: 1.78 A.

EXAMPLE 4.7

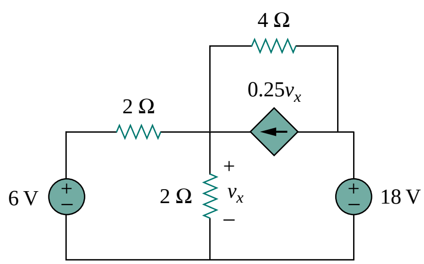


Figure 4.20 For Example 4.7.

Find v_x in Fig. 4.20 using source transformation.

Solution:

The circuit in Fig. 4.20 involves a voltage-controlled dependent current source. We transform this dependent current source as well as the 6-V independent voltage source as shown in Fig. 4.21(a). The 18-V voltage source is not transformed because it is not connected in series with any resistor. The two 2-Ω resistors in parallel combine to give a 1-Ω resistor, which is in parallel with the 3-A current source. The current is transformed to a voltage source as shown in Fig. 4.21(b). Notice that the terminals for v_x are intact. Applying KVL around the loop in Fig. 4.21(b) gives

$$-3 + 5i + v_x + 18 = 0 \tag{4.7.1}$$

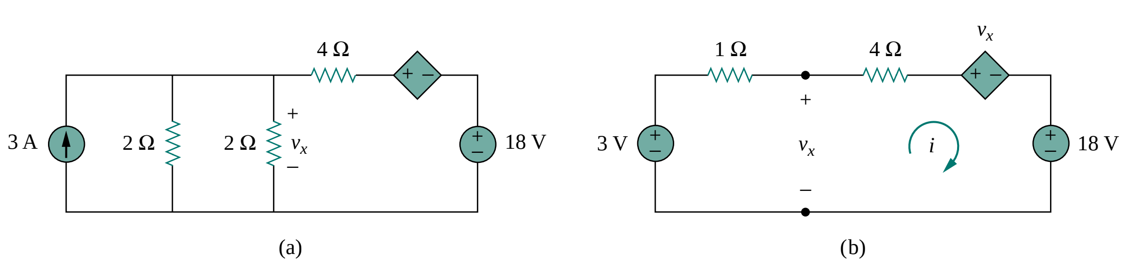


Figure 4.21 For Example 4.7: Applying source transformation to the circuit in Fig. 4.20.

Applying KVL to the loop containing only the 3-V voltage source, the 1-Ω resistor, and v_x yields

$$-3 + 1i + v_x = 0 \quad \Longrightarrow \quad v_x = 3 - i \tag{4.7.2}$$

Substituting this into Eq. (4.7.1), we obtain

$$15 + 5i + 3 - i = 0 \quad \Longrightarrow \quad i = -4.5 \text{ A}$$

Alternatively, we may apply KVL to the loop containing v_x, the 4-Ω resistor, the voltage-controlled dependent voltage source, and the 18-V voltage source in Fig. 4.21(b). We obtain

$$-v_x + 4i + v_x + 18 = 0 \quad \Longrightarrow \quad i = -4.5 \text{ A}$$

Thus, $v_x = 3 - i = 7.5$ V.

PRACTICE PROBLEM 4.7

Use source transformation to find i_x in the circuit shown in Fig. 4.22.

Answer: 1.176 A.

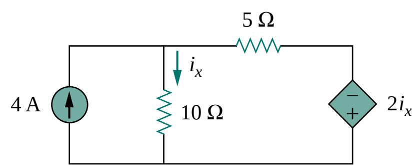


Figure 4.22 For Practice Prob. 4.7.

4.5 THEVENIN'S THEOREM

Electronic Testing Tutorials

It often occurs in practice that a particular element in a circuit is variable (usually called the *load*) while other elements are fixed. As a typical example, a household outlet terminal may be connected to different appliances constituting a variable load. Each time the variable element is changed, the entire circuit has to be analyzed all over again. To avoid this problem, Thevenin's theorem provides a technique by which the fixed part of the circuit is replaced by an equivalent circuit.

According to Thevenin's theorem, the linear circuit in Fig. 4.23(a) can be replaced by that in Fig. 4.23(b). (The load in Fig. 4.23 may be a single resistor or another circuit.) The circuit to the left of the terminals a-b in Fig. 4.23(b) is known as the *Thevenin equivalent circuit*; it was developed in 1883 by M. Leon Thevenin (1857–1926), a French telegraph engineer.

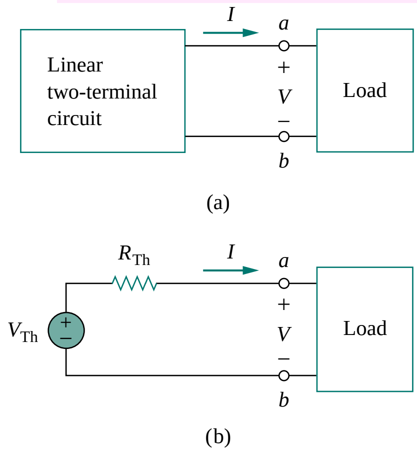


Figure 4.23 Replacing a linear two-terminal circuit by its Thevenin equivalent: (a) original circuit, (b) the Thevenin equivalent circuit.

> Thevenin's theorem states that a linear two-terminal circuit can be replaced by an equivalent circuit consisting of a voltage source V_{Th} in series with a resistor R_{Th}, where V_{Th} is the open-circuit voltage at the terminals and R_{Th} is the input or equivalent resistance at the terminals when the independent sources are turned off.

The proof of the theorem will be given later, in Section 4.7. Our major concern right now is how to find the Thevenin equivalent voltage

V_{Th} and resistance R_{Th}. To do so, suppose the two circuits in Fig. 4.23 are equivalent. Two circuits are said to be *equivalent* if they have the same voltage-current relation at their terminals. Let us find out what will make the two circuits in Fig. 4.23 equivalent. If the terminals a-b are made open-circuited (by removing the load), no current flows, so that the open-circuit voltage across the terminals a-b in Fig. 4.23(a) must be equal to the voltage source V_{Th} in Fig. 4.23(b), since the two circuits are equivalent. Thus V_{Th} is the open-circuit voltage across the terminals as shown in Fig. 4.24(a); that is,

$$V_{Th} = v_{oc} \tag{4.6}$$

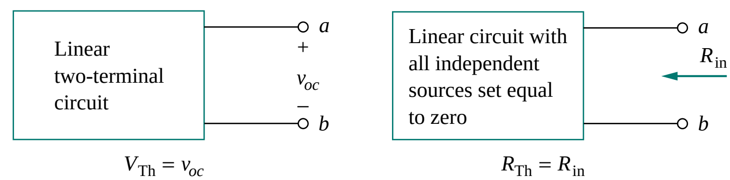


Figure 4.24 Finding V_{Th} and R_{Th}.

Again, with the load disconnected and terminals a-b open-circuited, we turn off all independent sources. The input resistance (or equivalent resistance) of the dead circuit at the terminals a-b in Fig. 4.23(a) must be equal to R_{Th} in Fig. 4.23(b) because the two circuits are equivalent. Thus, R_{Th} is the input resistance at the terminals when the independent sources are turned off, as shown in Fig. 4.24(b); that is,

$$R_{Th} = R_{in} \tag{4.7}$$

To apply this idea in finding the Thevenin resistance R_{Th}, we need to consider two cases.

CASE 1 If the network has no dependent sources, we turn off all independent sources. R_{Th} is the input resistance of the network looking between terminals a and b, as shown in Fig. 4.24(b).

CASE 2 If the network has dependent sources, we turn off all independent sources. As with superposition, dependent sources are not to be turned off because they are controlled by circuit variables. We apply a voltage source v_o at terminals a and b and determine the resulting current i_o. Then $R_{Th} = v_o/i_o$, as shown in Fig. 4.25(a). Alternatively, we may insert a current source i_o at terminals a-b as shown in Fig. 4.25(b) and find the terminal voltage v_o. Again $R_{Th} = v_o/i_o$. Either of the two approaches will give the same result. In either approach we may assume any value of v_o and i_o. For example, we may use $v_o = 1$ V or $i_o = 1$ A, or even use unspecified values of v_o or i_o.

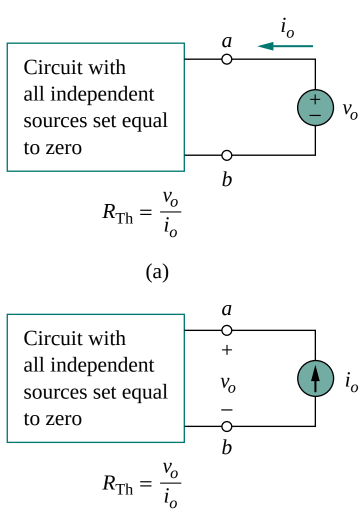


Figure 4.25 Finding R_{Th} when circuit has dependent sources.

Later we will see that an alternative way of finding R_{Th} is $R_{Th} = v_{oc}/i_{sc}$.

It often occurs that R_{Th} takes a negative value. In this case, the negative resistance ($v = -iR$) implies that the circuit is supplying power.

This is possible in a circuit with dependent sources; Example 4.10 will illustrate this.

Thevenin's theorem is very important in circuit analysis. It helps simplify a circuit. A large circuit may be replaced by a single independent voltage source and a single resistor. This replacement technique is a powerful tool in circuit design.

As mentioned earlier, a linear circuit with a variable load can be replaced by the Thevenin equivalent, exclusive of the load. The equivalent network behaves the same way externally as the original circuit. Consider a linear circuit terminated by a load R_L, as shown in Fig. 4.26(a). The current I_L through the load and the voltage V_L across the load are easily determined once the Thevenin equivalent of the circuit at the load's terminals is obtained, as shown in Fig. 4.26(b). From Fig. 4.26(b), we obtain

$$I_L = \frac{V_{\text{Th}}}{R_{\text{Th}} + R_L} \tag{4.8a}$$

$$V_L = R_L I_L = \frac{R_L}{R_{\text{Th}} + R_L} V_{\text{Th}} \tag{4.8b}$$

Note from Fig. 4.26(b) that the Thevenin equivalent is a simple voltage divider, yielding V_L by mere inspection.

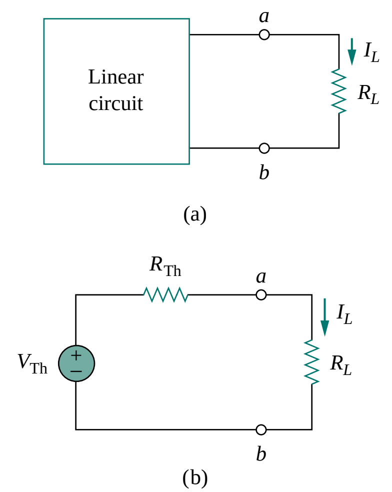


Figure 4.26 A circuit with a load: (a) original circuit, (b) Thevenin equivalent.

EXAMPLE 4.8

Find the Thevenin equivalent circuit of the circuit shown in Fig. 4.27, to the left of the terminals a-b. Then find the current through $R_L = 6$, 16, and 36 Ω.

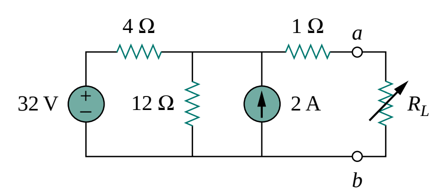


Figure 4.27 For Example 4.8.

Solution:

We find R_{Th} by turning off the 32-V voltage source (replacing it with a short circuit) and the 2-A current source (replacing it with an open circuit). The circuit becomes what is shown in Fig. 4.28(a). Thus,

$$R_{\text{Th}} = 4 \parallel 12 + 1 = \frac{4 \times 12}{16} + 1 = 4\ \Omega$$

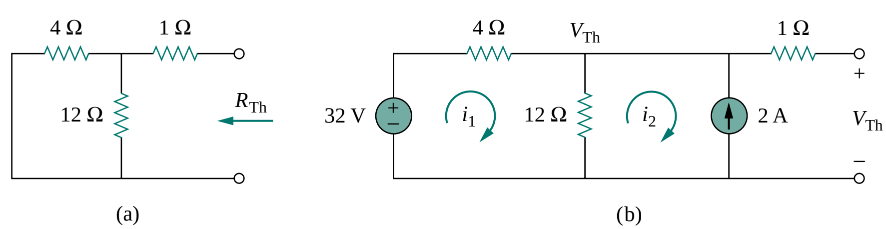


Figure 4.28 For Example 4.8: (a) finding R_{Th}, (b) finding V_{Th}.

To find V_{Th}, consider the circuit in Fig. 4.28(b). Applying mesh analysis to the two loops, we obtain

$$-32 + 4i_1 + 12(i_1 - i_2) = 0, \qquad i_2 = -2\text{ A}$$

Solving for i_1, we get $i_1 = 0.5$ A. Thus,

$$V_{Th} = 12(i_1 - i_2) = 12(0.5 + 2.0) = 30 \text{ V}$$

Alternatively, it is even easier to use nodal analysis. We ignore the 1-Ω resistor since no current flows through it. At the top node, KCL gives

$$\frac{32 - V_{Th}}{4} + 2 = \frac{V_{Th}}{12}$$

or

$$96 - 3V_{Th} + 24 = V_{Th} \quad \Longrightarrow \quad V_{Th} = 30 \text{ V}$$

as obtained before. We could also use source transformation to find V_{Th}.

The Thevenin equivalent circuit is shown in Fig. 4.29. The current through R_L is

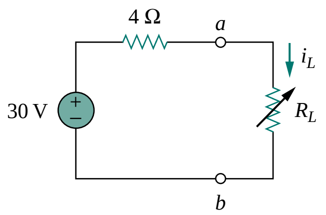


Figure 4.29 The Thevenin equivalent circuit for Example 4.8.

$$I_L = \frac{V_{Th}}{R_{Th} + R_L} = \frac{30}{4 + R_L}$$

When $R_L = 6$,

$$I_L = \frac{30}{10} = 3 \text{ A}$$

When $R_L = 16$,

$$I_L = \frac{30}{20} = 1.5 \text{ A}$$

When $R_L = 36$,

$$I_L = \frac{30}{40} = 0.75 \text{ A}$$

PRACTICE PROBLEM 4.8

Using Thevenin's theorem, find the equivalent circuit to the left of the terminals in the circuit in Fig. 4.30. Then find i.

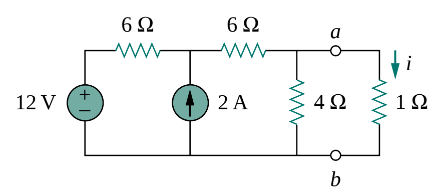


Figure 4.30 For Practice Prob. 4.8.

Answer: $V_{Th} = 6$ V, $R_{Th} = 3\ \Omega$, $i = 1.5$ A.

EXAMPLE 4.9

Find the Thevenin equivalent of the circuit in Fig. 4.31.

Solution:

This circuit contains a dependent source, unlike the circuit in the previous example. To find R_{Th}, we set the independent source equal to zero but leave the dependent source alone. Because of the presence of the dependent source, however, we excite the network with a voltage source v_o connected to the terminals as indicated in Fig. 4.32(a). We may set $v_o = 1$ V to ease calculation, since the circuit is linear. Our goal is to find the current i_o through the terminals, and then obtain $R_{Th} = 1/i_o$. (Alternatively, we may insert a 1-A current source, find the corresponding voltage v_o, and obtain $R_{Th} = v_o/1$.)

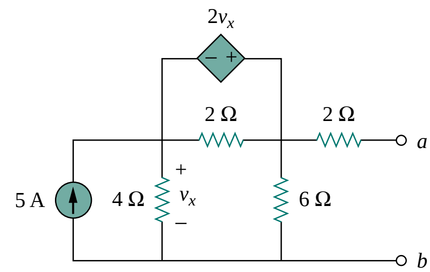


Figure 4.31 For Example 4.9.

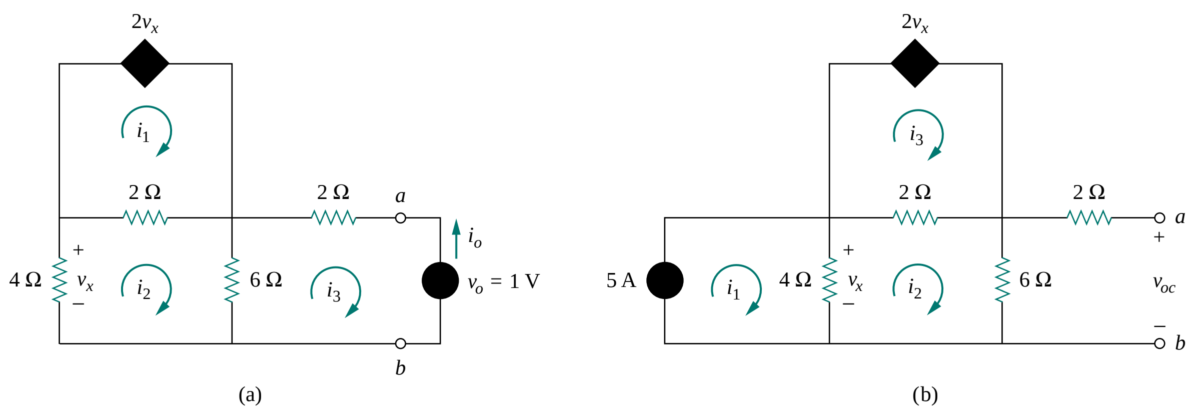


Figure 4.32 Finding R_{Th} and V_{Th} for Example 4.9.

Applying mesh analysis to loop 1 in the circuit in Fig. 4.32(a) results in

$$-2v_x + 2(i_1 - i_2) = 0 \qquad \text{or} \qquad v_x = i_1 - i_2$$

But $-4i_2 = v_x = i_1 - i_2$; hence,

$$i_1 = -3i_2 \tag{4.9.1}$$

For loops 2 and 3, applying KVL produces

$$4i_2 + 2(i_2 - i_1) + 6(i_2 - i_3) = 0 \tag{4.9.2}$$

$$6(i_3 - i_2) + 2i_3 + 1 = 0 \tag{4.9.3}$$

Solving these equations gives

$$i_3 = -\frac{1}{6}\text{ A}$$

But $i_o = -i_3 = 1/6$ A. Hence,

$$R_{Th} = \frac{1\text{ V}}{i_o} = 6\ \Omega$$

To get V_{Th}, we find v_{oc} in the circuit of Fig. 4.32(b). Applying mesh analysis, we get

$$i_1 = 5 \tag{4.9.4}$$

$$-2v_x + 2(i_3 - i_2) = 0 \quad \Longrightarrow \quad v_x = i_3 - i_2 \tag{4.9.5}$$

$$4(i_2 - i_1) + 2(i_2 - i_3) + 6i_2 = 0$$

or

$$12i_2 - 4i_1 - 2i_3 = 0 \tag{4.9.6}$$

But $4(i_1 - i_2) = v_x$. Solving these equations leads to $i_2 = 10/3$. Hence,

$$V_{\text{Th}} = v_{oc} = 6i_2 = 20\ \text{V}$$

The Thevenin equivalent is as shown in Fig. 4.33.

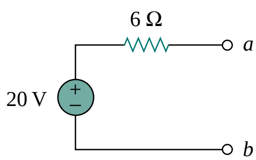


Figure 4.33 The Thevenin equivalent of the circuit in Fig. 4.31.

PRACTICE PROBLEM 4.9

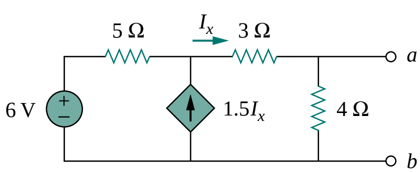


Figure 4.34 For Practice Prob. 4.9.

Find the Thevenin equivalent circuit of the circuit in Fig. 4.34 to the left of the terminals.

Answer: $V_{\text{Th}} = 5.33$ V, $R_{\text{Th}} = 0.44\ \Omega$.

EXAMPLE 4.10

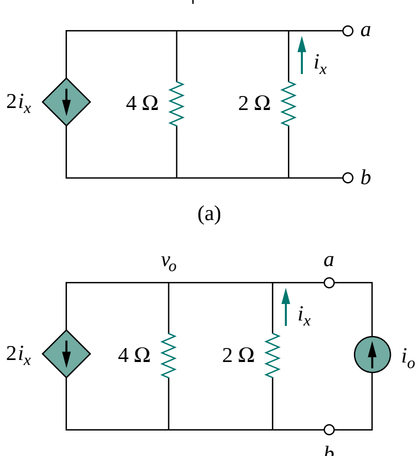


Figure 4.35 For Example 4.10.

Determine the Thevenin equivalent of the circuit in Fig. 4.35(a).

Solution:

Since the circuit in Fig. 4.35(a) has no independent sources, $V_{\text{Th}} = 0$ V. To find R_{Th}, it is best to apply a current source i_o at the terminals as shown in Fig. 4.35(b). Applying nodal analysis gives

$$i_o + i_x = 2i_x + \frac{v_o}{4} \tag{4.10.1}$$

But

$$i_x = \frac{0 - v_o}{2} = -\frac{v_o}{2} \tag{4.10.2}$$

Substituting Eq. (4.10.2) into Eq. (4.10.1) yields

$$i_o = i_x + \frac{v_o}{4} = -\frac{v_o}{2} + \frac{v_o}{4} = -\frac{v_o}{4} \qquad \text{or} \qquad v_o = -4i_o$$

Thus,

$$R_{\text{Th}} = \frac{v_o}{i_o} = -4\ \Omega$$

The negative value of the resistance tells us that, according to the passive sign convention, the circuit in Fig. 4.35(a) is supplying power. Of course, the resistors in Fig. 4.35(a) cannot supply power (they absorb power); it

is the dependent source that supplies the power. This is an example of how a dependent source and resistors could be used to simulate negative resistance.

Network Analysis

PRACTICE PROBLEM 4.10

Obtain the Thevenin equivalent of the circuit in Fig. 4.36.

Answer: $V_{Th} = 0$ V, $R_{Th} = -7.5\ \Omega$.

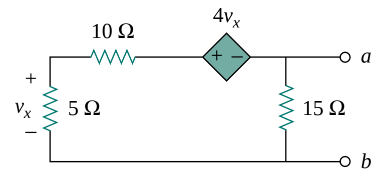


Figure 4.36 For Practice Prob. 4.10.

4.6 NORTON'S THEOREM

Electronic Testing Tutorials

In 1926, about 43 years after Thevenin published his theorem, E. L. Norton, an American engineer at Bell Telephone Laboratories, proposed a similar theorem.

> **Norton's theorem** states that a linear two-terminal circuit can be replaced by an equivalent circuit consisting of a current source I_N in parallel with a resistor R_N, where I_N is the short-circuit current through the terminals and R_N is the input or equivalent resistance at the terminals when the independent sources are turned off.

Thus, the circuit in Fig. 4.37(a) can be replaced by the one in Fig. 4.37(b).

The proof of Norton's theorem will be given in the next section. For now, we are mainly concerned with how to get R_N and I_N. We find R_N in the same way we find R_{Th}. In fact, from what we know about source transformation, the Thevenin and Norton resistances are equal; that is,

$$\boxed{R_N = R_{Th}} \tag{4.9}$$

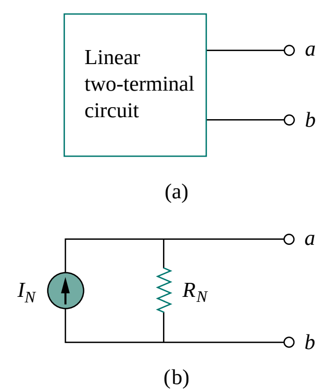


Figure 4.37 (a) Original circuit, (b) Norton equivalent circuit.

To find the Norton current I_N, we determine the short-circuit current flowing from terminal a to b in both circuits in Fig. 4.37. It is evident that the short-circuit current in Fig. 4.37(b) is I_N. This must be the same short-circuit current from terminal a to b in Fig. 4.37(a), since the two circuits are equivalent. Thus,

$$I_N = i_{sc} \tag{4.10}$$

shown in Fig. 4.38. Dependent and independent sources are treated the same way as in Thevenin's theorem.

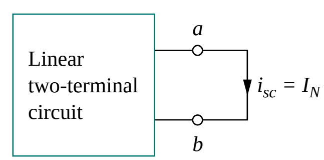


Figure 4.38 Finding Norton current I_N.

Observe the close relationship between Norton's and Thevenin's theorems: $R_N = R_{Th}$ as in Eq. (4.9), and

$$\boxed{I_N = \frac{V_{Th}}{R_{Th}}} \tag{4.11}$$

The Thevenin and Norton equivalent circuits are related by a source transformation.

This is essentially source transformation. For this reason, source transformation is often called Thevenin-Norton transformation.

Since V_{Th}, I_N, and R_{Th} are related according to Eq. (4.11), to determine the Thevenin or Norton equivalent circuit requires that we find:

- The open-circuit voltage v_{oc} across terminals a and b.
- The short-circuit current i_{sc} at terminals a and b.
- The equivalent or input resistance R_{in} at terminals a and b when all independent sources are turned off.

We can calculate any two of the three using the method that takes the least effort and use them to get the third using Ohm's law. Example 4.11 will illustrate this. Also, since

$$V_{\text{Th}} = v_{oc} \tag{4.12a}$$

$$I_N = i_{sc} \tag{4.12b}$$

$$R_{\text{Th}} = \frac{v_{oc}}{i_{sc}} = R_N \tag{4.12c}$$

the open-circuit and short-circuit tests are sufficient to find any Thevenin or Norton equivalent.

EXAMPLE 4.11

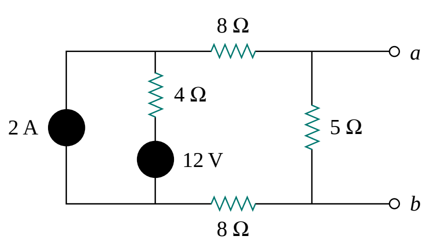


Figure 4.39 For Example 4.11.

Find the Norton equivalent circuit of the circuit in Fig. 4.39.

Solution:

We find R_N in the same way we find R_{Th} in the Thevenin equivalent circuit. Set the independent sources equal to zero. This leads to the circuit in Fig. 4.40(a), from which we find R_N. Thus,

$$R_N = 5 \parallel (8 + 4 + 8) = 5 \parallel 20 = \frac{20 \times 5}{25} = 4\ \Omega$$

To find I_N, we short-circuit terminals a and b, as shown in Fig. 4.40(b). We ignore the 5-Ω resistor because it has been short-circuited. Applying mesh analysis, we obtain

$$i_1 = 2\text{ A}, \qquad 20i_2 - 4i_1 - 12 = 0$$

From these equations, we obtain

$$i_2 = 1\text{ A} = i_{sc} = I_N$$

Alternatively, we may determine I_N from $V_{\text{Th}}/R_{\text{Th}}$. We obtain V_{Th} as the open-circuit voltage across terminals a and b in Fig. 4.40(c). Using mesh analysis, we obtain

$$\begin{aligned} i_3 &= 2\text{ A} \\ 25i_4 - 4i_3 - 12 &= 0 \qquad \Longrightarrow \qquad i_4 = 0.8\text{ A} \end{aligned}$$

and

$$v_{oc} = V_{\text{Th}} = 5i_4 = 4\text{ V}$$

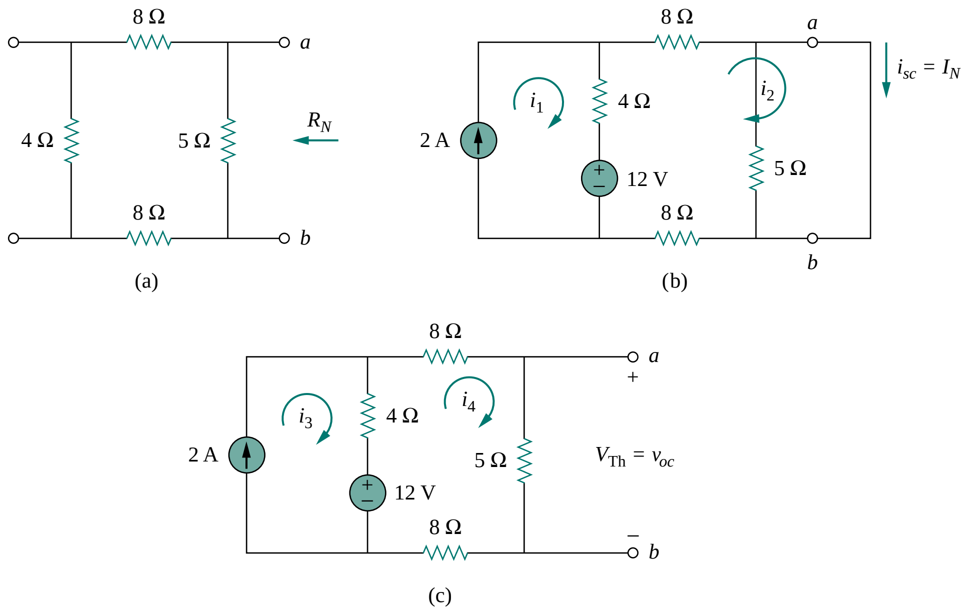


Figure 4.40 For Example 4.11; finding: (a) R_N, (b) $I_N = i_{sc}$, (c) $V_{Th} = v_{oc}$.

Hence,

$$I_N = \frac{V_{Th}}{R_{Th}} = \frac{4}{4} = 1 \text{ A}$$

as obtained previously. This also serves to confirm Eq. (4.7) that $R_{Th} = v_{oc}/i_{sc} = 4/1 = 4\ \Omega$. Thus, the Norton equivalent circuit is as shown in Fig. 4.41.

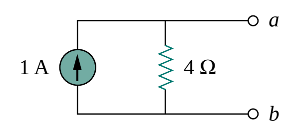


Figure 4.41 Norton equivalent of the circuit in Fig. 4.39.

PRACTICE PROBLEM 4.11

Find the Norton equivalent circuit for the circuit in Fig. 4.42.

Answer: $R_N = 3\ \Omega$, $I_N = 4.5$ A.

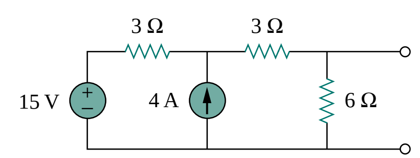


Figure 4.42 For Practice Prob. 4.11.

EXAMPLE 4.12

Using Norton's theorem, find R_N and I_N of the circuit in Fig. 4.43 at terminals a-b.

Solution:

To find R_N, we set the independent voltage source equal to zero and connect a voltage source of $v_o = 1$ V (or any unspecified voltage v_o) to the

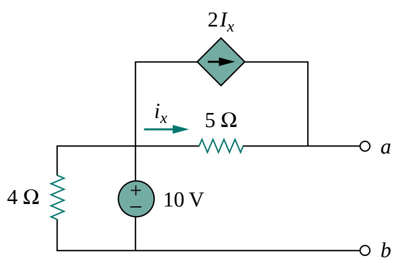


Figure 4.43 For Example 4.12.

terminals. We obtain the circuit in Fig. 4.44(a). We ignore the 4-Ω resistor because it is short-circuited. Also due to the short circuit, the 5-Ω resistor, the voltage source, and the dependent current source are all in parallel. Hence, $i_x = v_o/5 = 1/5 = 0.2$. At node a, $-i_o = i_x + 2i_x = 3i_x = 0.6$, and

$$R_N = \frac{v_o}{i_o} = \frac{1}{-0.6} = -1.67\ \Omega$$

To find I_N, we short-circuit terminals a and b and find the current i_{sc}, as indicated in Fig. 4.44(b). Note from this figure that the 4-Ω resistor, the 10-V voltage source, the 5-Ω resistor, and the dependent current source are all in parallel. Hence,

$$i_x = \frac{10 - 0}{5} = 2\ \text{A}$$

At node a, KCL gives

$$i_{sc} = i_x + 2i_x = 2 + 4 = 6\ \text{A}$$

Thus,

$$I_N = 6\ \text{A}$$

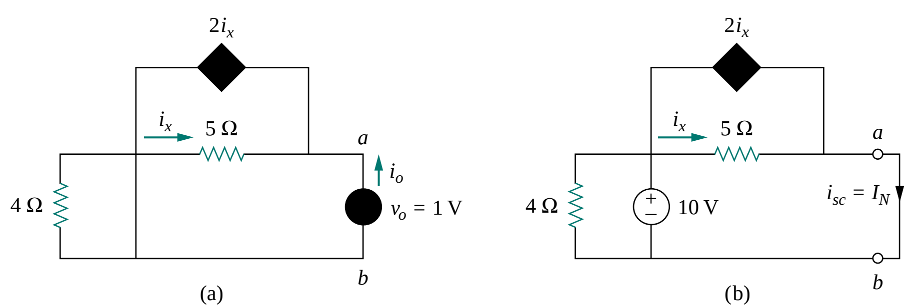


Figure 4.44 For Example 4.12: (a) finding R_N, (b) finding I_N.

PRACTICE PROBLEM 4.12

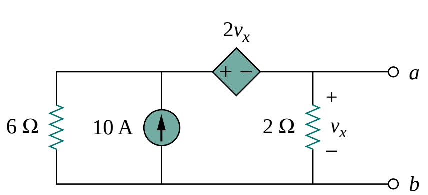


Figure 4.45 For Practice Prob. 4.12.

Find the Norton equivalent circuit of the circuit in Fig. 4.45.

Answer: $R_N = 1\ \Omega$, $I_N = 10$ A.

†4.7 DERIVATIONS OF THEVENIN'S AND NORTON'S THEOREMS

In this section, we will prove Thevenin's and Norton's theorems using the superposition principle.

Consider the linear circuit in Fig. 4.46(a). It is assumed that the circuit contains resistors, and dependent and independent sources. We have access to the circuit via terminals a and b, through which current from an external source is applied. Our objective is to ensure that the voltage-current relation at terminals a and b is identical to that of the Thevenin equivalent in Fig. 4.46(b). For the sake of simplicity, suppose the linear circuit in Fig. 4.46(a) contains two independent voltage sources v_{s1} and v_{s2} and two independent current sources i_{s1} and i_{s2}. We may obtain any circuit variable, such as the terminal voltage v, by applying superposition. That is, we consider the contribution due to each independent source including the external source i. By superposition, the terminal voltage v is

$$v = A_0 i + A_1 v_{s1} + A_2 v_{s2} + A_3 i_{s1} + A_4 i_{s2} \tag{4.13}$$

where A_0, A_1, A_2, A_3, and A_4 are constants. Each term on the right-hand side of Eq. (4.13) is the contribution of the related independent source; that is, $A_0 i$ is the contribution to v due to the external current source i, $A_1 v_{s1}$ is the contribution due to the voltage source v_{s1}, and so on. We may collect terms for the internal independent sources together as B_0, so that Eq. (4.13) becomes

$$v = A_0 i + B_0 \tag{4.14}$$

where $B_0 = A_1 v_{s1} + A_2 v_{s2} + A_3 i_{s1} + A_4 i_{s2}$. We now want to evaluate the values of constants A_0 and B_0. When the terminals a and b are open-circuited, $i = 0$ and $v = B_0$. Thus B_0 is the open-circuit voltage v_{oc}, which is the same as V_{Th}, so

$$B_0 = V_{\text{Th}} \tag{4.15}$$

When all the internal sources are turned off, $B_0 = 0$. The circuit can then be replaced by an equivalent resistance R_{eq}, which is the same as R_{Th}, and Eq. (4.14) becomes

$$v = A_0 i = R_{\text{Th}} i \qquad \Longrightarrow \qquad A_0 = R_{\text{Th}} \tag{4.16}$$

Substituting the values of A_0 and B_0 in Eq. (4.14) gives

$$v = R_{\text{Th}} i + V_{\text{Th}} \tag{4.17}$$

which expresses the voltage-current relation at terminals a and b of the circuit in Fig. 4.46(b). Thus, the two circuits in Fig. 4.46(a) and 4.46(b) are equivalent.

When the same linear circuit is driven by a voltage source v as shown in Fig. 4.47(a), the current flowing into the circuit can be obtained by superposition as

$$i = C_0 v + D_0 \tag{4.18}$$

where $C_0 v$ is the contribution to i due to the external voltage source v and D_0 contains the contributions to i due to all internal independent sources. When the terminals a-b are short-circuited, $v = 0$ so that $i = D_0 = -i_{sc}$, where i_{sc} is the short-circuit current flowing out of terminal a, which is the same as the Norton current I_N, i.e.,

$$D_0 = -I_N \tag{4.19}$$

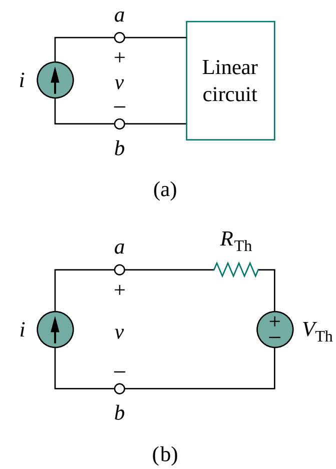


Figure 4.46 Derivation of Thevenin equivalent: (a) a current-driven circuit, (b) its Thevenin equivalent.

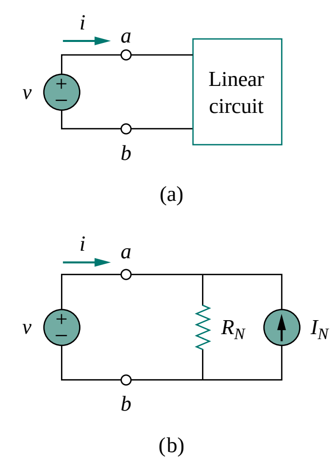


Figure 4.47 Derivation of Norton equivalent: (a) a voltage-driven circuit, (b) its Norton equivalent.

When all the internal independent sources are turned off, $D_0 = 0$ and the circuit can be replaced by an equivalent resistance R_{eq} (or an equivalent conductance $G_{eq} = 1/R_{eq}$), which is the same as R_{Th} or R_N. Thus Eq. (4.19) becomes

$$i = \frac{v}{R_{Th}} - I_N \tag{4.20}$$

This expresses the voltage-current relation at terminals *a-b* of the circuit in Fig. 4.47(b), confirming that the two circuits in Fig. 4.47(a) and 4.47(b) are equivalent.

4.8 MAXIMUM POWER TRANSFER

In many practical situations, a circuit is designed to provide power to a load. While for electric utilities, minimizing power losses in the process of transmission and distribution is critical for efficiency and economic reasons, there are other applications in areas such as communications where it is desirable to maximize the power delivered to a load. We now address the problem of delivering the maximum power to a load when given a system with known internal losses. It should be noted that this will result in significant internal losses greater than or equal to the power delivered to the load.

The Thevenin equivalent is useful in finding the maximum power a linear circuit can deliver to a load. We assume that we can adjust the load resistance R_L. If the entire circuit is replaced by its Thevenin equivalent except for the load, as shown in Fig. 4.48, the power delivered to the load is

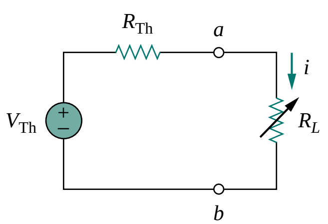


Figure 4.48 The circuit used for maximum power transfer.

$$p = i^2 R_L = \left(\frac{V_{Th}}{R_{Th} + R_L}\right)^2 R_L \tag{4.21}$$

For a given circuit, V_{Th} and R_{Th} are fixed. By varying the load resistance R_L, the power delivered to the load varies as sketched in Fig. 4.49. We notice from Fig. 4.49 that the power is small for small or large values of R_L but maximum for some value of R_L between 0 and ∞. We now want to show that this maximum power occurs when R_L is equal to R_{Th}. This is known as the *maximum power theorem.*

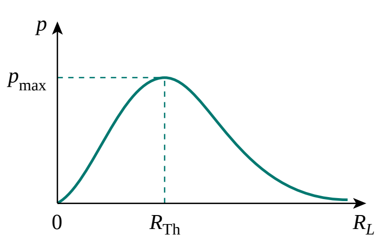


Figure 4.49 Power delivered to the load as a function of R_L.

> **Maximum power** is transferred to the load when the load resistance equals the Thevenin resistance as seen from the load ($R_L = R_{Th}$).

To prove the maximum power transfer theorem, we differentiate p in Eq. (4.21) with respect to R_L and set the result equal to zero. We obtain

$$\begin{aligned}\frac{dp}{dR_L} &= V_{Th}^2 \left[\frac{(R_{Th} + R_L)^2 - 2R_L(R_{Th} + R_L)}{(R_{Th} + R_L)^4}\right] \\ &= V_{Th}^2 \left[\frac{(R_{Th} + R_L - 2R_L)}{(R_{Th} + R_L)^3}\right] = 0\end{aligned}$$

This implies that

$$0 = (R_{\text{Th}} + R_L - 2R_L) = (R_{\text{Th}} - R_L) \tag{4.22}$$

which yields

$$\boxed{R_L = R_{\text{Th}}} \tag{4.23}$$

showing that the maximum power transfer takes place when the load resistance R_L equals the Thevenin resistance R_{Th}. We can readily confirm that Eq. (4.23) gives the maximum power by showing that $d^2p/dR_L^2 < 0$.

The source and load are said to be *matched* when $R_L = R_{\text{Th}}$.

The maximum power transferred is obtained by substituting Eq. (4.23) into Eq. (4.21), for

$$\boxed{p_{\max} = \frac{V_{\text{Th}}^2}{4R_{\text{Th}}}} \tag{4.24}$$

Equation (4.24) applies only when $R_L = R_{\text{Th}}$. When $R_L \neq R_{\text{Th}}$, we compute the power delivered to the load using Eq. (4.21).

EXAMPLE 4.13

Find the value of R_L for maximum power transfer in the circuit of Fig. 4.50. Find the maximum power.

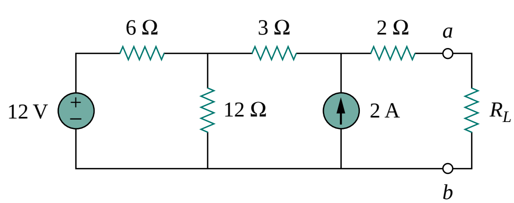


Figure 4.50 For Example 4.13.

Solution:

We need to find the Thevenin resistance R_{Th} and the Thevenin voltage V_{Th} across the terminals a-b. To get R_{Th}, we use the circuit in Fig. 4.51(a) and obtain

$$R_{\text{Th}} = 2 + 3 + 6 \parallel 12 = 5 + \frac{6 \times 12}{18} = 9\ \Omega$$

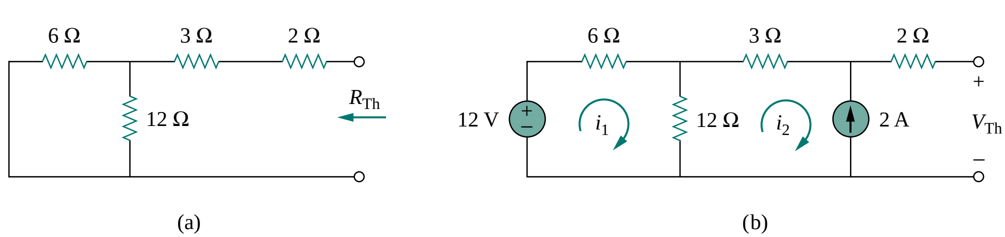


Figure 4.51 For Example 4.13: (a) finding R_{Th}, (b) finding V_{Th}.

To get V_{Th}, we consider the circuit in Fig. 4.51(b). Applying mesh analysis,

$$-12 + 18i_1 - 12i_2 = 0, \qquad i_2 = -2 \text{ A}$$

Solving for i_1, we get $i_1 = -2/3$. Applying KVL around the outer loop to get V_{Th} across terminals a-b, we obtain

$$-12 + 6i_1 + 3i_2 + 2(0) + V_{\mathrm{Th}} = 0 \qquad \Longrightarrow \qquad V_{\mathrm{Th}} = 22 \text{ V}$$

For maximum power transfer,

$$R_L = R_{\mathrm{Th}} = 9\ \Omega$$

and the maximum power is

$$p_{\max} = \frac{V_{\mathrm{Th}}^2}{4R_L} = \frac{22^2}{4 \times 9} = 13.44 \text{ W}$$

PRACTICE PROBLEM 4.13

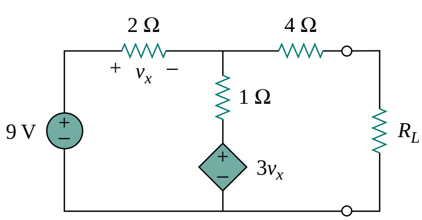


Figure 4.52 For Practice Prob. 4.13.

Determine the value of R_L that will draw the maximum power from the rest of the circuit in Fig. 4.52. Calculate the maximum power.

Answer: 4.22 Ω, 2.901 W.

4.9 VERIFYING CIRCUIT THEOREMS WITH *PSPICE*

In this section, we learn how to use *PSpice* to verify the theorems covered in this chapter. Specifically, we will consider using dc sweep analysis to find the Thevenin or Norton equivalent at any pair of nodes in a circuit and the maximum power transfer to a load. The reader is advised to read Section D.3 of Appendix D in preparation for this section.

To find the Thevenin equivalent of a circuit at a pair of open terminals using *PSpice*, we use the schematic editor to draw the circuit and insert an independent probing current source, say, Ip, at the terminals. The probing current source must have a part name ISRC. We then perform a DC Sweep on Ip, as discussed in Section D.3. Typically, we may let the current through Ip vary from 0 to 1 A in 0.1-A increments. After simulating the circuit, we use Probe to display a plot of the voltage across Ip versus the current through Ip. The zero intercept of the plot gives us the Thevenin equivalent voltage, while the slope of the plot is equal to the Thevenin resistance.

To find the Norton equivalent involves similar steps except that we insert a probing independent voltage source (with a part name VSRC), say, Vp, at the terminals. We perform a DC Sweep on Vp and let Vp vary from 0 to 1 V in 0.1-V increments. A plot of the current through

Vp versus the voltage across Vp is obtained using the Probe menu after simulation. The zero intercept is equal to the Norton current, while the slope of the plot is equal to the Norton conductance.

To find the maximum power transfer to a load using *PSpice* involves performing a dc parametric sweep on the component value of R_L in Fig. 4.48 and plotting the power delivered to the load as a function of R_L. According to Fig. 4.49, the maximum power occurs when $R_L = R_{\text{Th}}$. This is best illustrated with an example, and Example 4.15 provides one.

We use VSRC and ISRC as part names for the independent voltage and current sources.

EXAMPLE 4.14

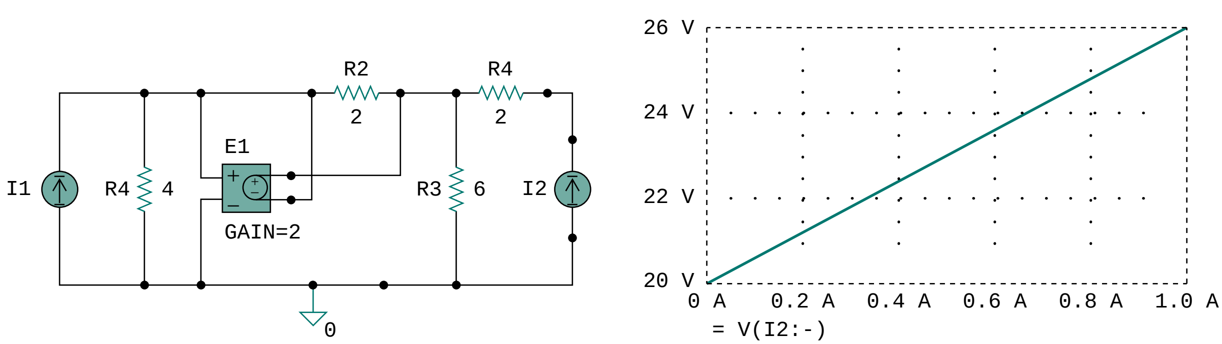
Consider the circuit is in Fig. 4.31 (see Example 4.9). Use *PSpice* to find the Thevenin and Norton equivalent circuits.

Solution:

(a) To find the Thevenin resistance R_{Th} and Thevenin voltage V_{Th} at the terminals a-b in the circuit in Fig. 4.31, we first use Schematics to draw the circuit as shown in Fig. 4.53(a). Notice that a probing current source I2 is inserted at the terminals. Under **Analysis/Setput**, we select DC Sweep. In the DC Sweep dialog box, we select Linear for the *Sweep Type* and Current Source for the *Sweep Var. Type*. We enter I2 under the *Name* box, 0 as *Start Value*, 1 as *End Value*, and 0.1 as *Increment*. After simulation, we add trace V(I2:−) from the Probe menu and obtain the plot shown in Fig. 4.53(b). From the plot, we obtain

$$V_{\text{Th}} = \text{Zero intercept} = 20 \text{ V}, \qquad R_{\text{Th}} = \text{Slope} = \frac{26-20}{1} = 6\ \Omega$$

These agree with what we got analytically in Example 4.9.



(a) (b)

Figure 4.53 For Example 4.14: (a) schematic and (b) plot for finding R_{Th} and V_{Th}.

(b) To find the Norton equivalent, we modify the schematic in Fig. 4.53(a) by replaying the probing current source with a probing voltage source V1. The result is the schematic in Fig. 4.54(a). Again, in the DC Sweep dialog box, we select Linear for the *Sweep Type* and Voltage Source for the *Sweep Var. Type*. We enter V1 under *Name* box, 0 as *Start Value*, 1 as *End Value*,

and 0.1 as *Increment*. When the Probe is running, we add trace I(V1) and obtain the plot in Fig. 4.54(b). From the plot, we obtain

$$I_N = \text{Zero intercept} = 3.335 \text{ A}$$

$$G_N = \text{Slope} = \frac{3.335 - 3.165}{1} = 0.17 \text{ S}$$

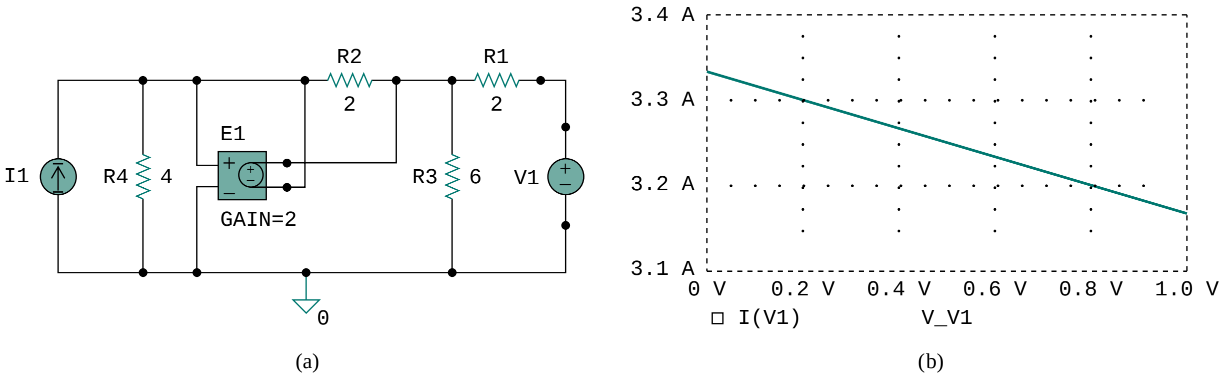


Figure 4.54 For Example 4.14: (a) schematic and (b) plot for finding G_N and I_N.

PRACTICE PROBLEM 4.14

Rework Practice Prob. 4.9 using *PSpice*.

Answer: $V_{\text{Th}} = 5.33$ V, $R_{\text{Th}} = 0.44\ \Omega$.

EXAMPLE 4.15

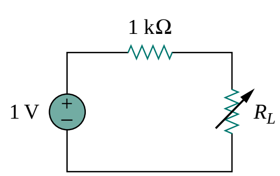


Figure 4.55 For Example 4.15.

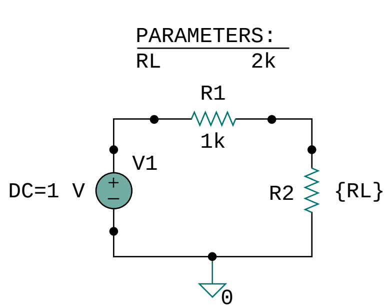


Figure 4.56 Schematic for the circuit in Fig. 4.55.

Refer to the circuit in Fig. 4.55. Use *PSpice* to find the maximum power transfer to R_L.

Solution:

We need to perform a dc sweep on R_L to determine when the power across it is maximum. We first draw the circuit using Schematics as shown in Fig. 4.56. Once the circuit is drawn, we take the following three steps to further prepare the circuit for a dc sweep.

The first step involves defining the value of R_L as a parameter, since we want to vary it. To do this:

1. **DCLICKL** the value 1k of R2 (representing R_L) to open up the *Set Attribute Value* dialog box.
2. Replace 1k with {RL} and click **OK** to accept the change.

Note that the curly brackets are necessary.

The second step is to define parameter. To achieve this:

1. Select **Draw/Get New Part/Libraries** · · ·**/special.slb**.
2. Type PARAM in the *PartName* box and click **OK**.
3. **DRAG** the box to any position near the circuit.
4. **CLICKL** to end placement mode.

5. **DCLICKL** to open up the *PartName: PARAM* dialog box.
6. **CLICKL** on *NAME1* = and enter RL (with no curly brackets) in the *Value* box, and **CLICKL Save Attr** to accept change.
7. **CLICKL** on *VALUE1* = and enter 2k in the *Value* box, and **CLICKL Save Attr** to accept change.
8. Click **OK**.

The value 2k in item 7 is necessary for a bias point calculation; it cannot be left blank.

The third step is to set up the DC Sweep to sweep the parameter. To do this:

1. Select **Analysis/Setput** to bring up the DC Sweep dialog box.
2. For the *Sweep Type*, select Linear (or Octave for a wide range of R_L).
3. For the *Sweep Var. Type*, select Global Parameter.
4. Under the *Name* box, enter RL.
5. In the *Start Value* box, enter 100.
6. In the *End Value* box, enter 5k.
7. In the *Increment* box, enter 100.
8. Click **OK** and **Close** to accept the parameters.

After taking these steps and saving the circuit, we are ready to simulate. Select **Analysis/Simulate**. If there are no errors, we select **Add Trace** in the Probe menu and type −V(R2:2)*I(R2) in the *Trace Command* box. [The negative sign is needed since I(R2) is negative.] This gives the plot of the power delivered to R_L as R_L varies from 100 Ω to 5 kΩ. We can also obtain the power absorbed by R_L by typing V(R2:2)*V(R2:2)/RL in the *Trace Command* box. Either way, we obtain the plot in Fig. 4.57. It is evident from the plot that the maximum power is 250 μW. Notice that the maximum occurs when $R_L = 1$ kΩ, as expected analytically.

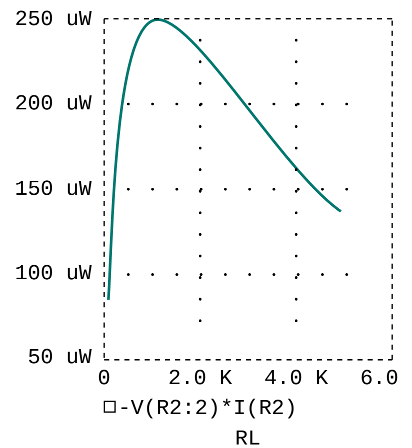


Figure 4.57 For Example 4.15: the plot of power across P_L.

PRACTICE PROBLEM 4.15

Find the maximum power transferred to R_L if the 1-kΩ resistor in Fig. 4.55 is replaced by a 2-kΩ resistor.

Answer: 125 μW.

†4.10 APPLICATIONS

In this section we will discuss two important practical applications of the concepts covered in this chapter: source modeling and resistance measurement.

4.10.1 Source Modeling

Source modeling provides an example of the usefulness of the Thevenin or the Norton equivalent. An active source such as a battery is often characterized by its Thevenin or Norton equivalent circuit. An ideal voltage source provides a constant voltage irrespective of the current

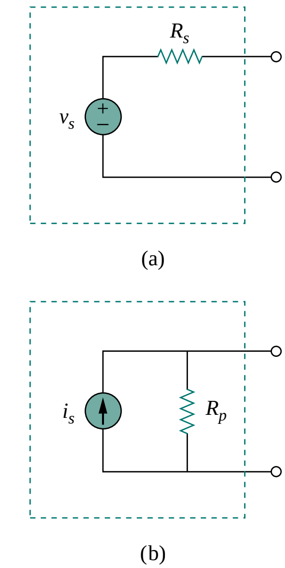


Figure 4.58 (a) Practical voltage source, (b) practical current source.

drawn by the load, while an ideal current source supplies a constant current regardless of the load voltage. As Fig. 4.58 shows, practical voltage and current sources are not ideal, due to their *internal resistances* or *source resistances* R_s and R_p. They become ideal as $R_s \to 0$ and $R_p \to \infty$. To show that this is the case, consider the effect of the load on voltage sources, as shown in Fig. 4.59(a). By the voltage division principle, the load voltage is

$$v_L = \frac{R_L}{R_s + R_L} v_s \tag{4.25}$$

As R_L increases, the load voltage approaches a source voltage v_s, as illustrated in Fig. 4.59(b). From Eq. (4.25), we should note that:

1. The load voltage will be constant if the internal resistance R_s of the source is zero or, at least, $R_s \ll R_L$. In other words, the smaller R_s is compared to R_L, the closer the voltage source is to being ideal.
2. When the load is disconnected (i.e., the source is open-circuited so that $R_L \to \infty$), $v_{oc} = v_s$. Thus, v_s may be regarded as the *unloaded source* voltage. The connection of the load causes the terminal voltage to drop in magnitude; this is known as the *loading effect*.

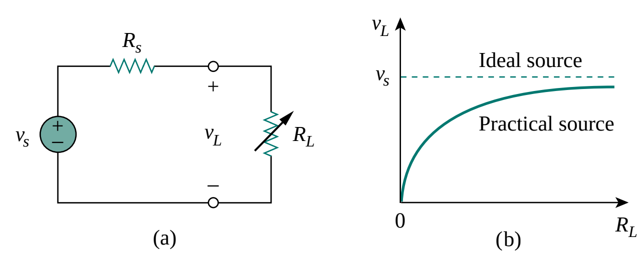


Figure 4.59 (a) Practical voltage source connected to a load R_L, (b) load voltage decreases as R_L decreases.

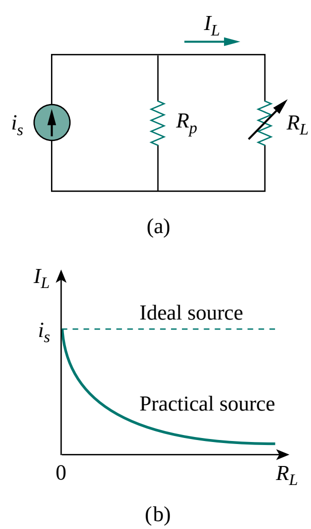


Figure 4.60 (a) Practical current source connected to a load R_L, (b) load current decreases as R_L increases.

The same argument can be made for a practical current source when connected to a load as shown in Fig. 4.60(a). By the current division principle,

$$i_L = \frac{R_p}{R_p + R_L} i_s \tag{4.26}$$

Figure 4.60(b) shows the variation in the load current as the load resistance increases. Again, we notice a drop in current due to the load (loading effect), and load current is constant (ideal current source) when the internal resistance is very large (i.e., $R_p \to \infty$ or, at least, $R_p \gg R_L$).

Sometimes, we need to know the unloaded source voltage v_s and the internal resistance R_s of a voltage source. To find v_s and R_s, we follow the procedure illustrated in Fig. 4.61. First, we measure the open-circuit voltage v_{oc} as in Fig. 4.61(a) and set

$$v_s = v_{oc} \tag{4.27}$$

Then, we connect a variable load R_L across the terminals as in Fig. 4.61(b). We adjust the resistance R_L until we measure a load voltage of exactly one-half of the open-circuit voltage, $v_L = v_{oc}/2$, because now $R_L = R_{\text{Th}} = R_s$. At that point, we disconnect R_L and measure it. We set

$$R_s = R_L \tag{4.28}$$

For example, a car battery may have $v_s = 12$ V and $R_s = 0.05\ \Omega$.

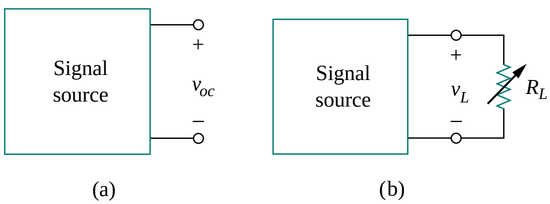


Figure 4.61 (a) Measuring v_{oc}, (b) measuring v_L.

EXAMPLE 4.16

The terminal voltage of a voltage source is 12 V when connected to a 2-W load. When the load is disconnected, the terminal voltage rises to 12.4 V. (a) Calculate the source voltage v_s and internal resistance R_s. (b) Determine the voltage when an 8-Ω load is connected to the source.

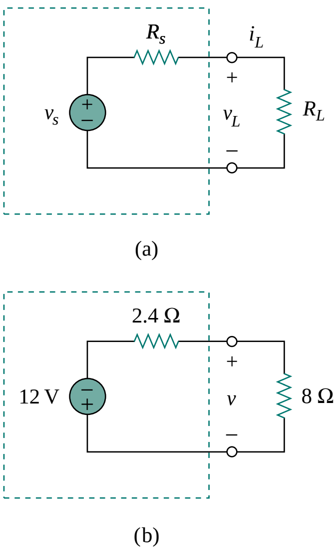


Figure 4.62 For Example 4.16.

Solution:

(a) We replace the source by its Thevenin equivalent. The terminal voltage when the load is disconnected is the open-circuit voltage,

$$v_s = v_{oc} = 12.4 \text{ V}$$

When the load is connected, as shown in Fig. 4.62(a), $v_L = 12$ V and $p_L = 2$ W. Hence,

$$p_L = \frac{vL^2}{R_L} \qquad \Longrightarrow \qquad R_L = \frac{v_L^2}{p_L} = \frac{12^2}{2} = 72\ \Omega$$

The load current is

$$i_L = \frac{v_L}{R_L} = \frac{12}{72} = \frac{1}{6} \text{ A}$$

The voltage across R_s is the difference between the source voltage v_s and the load voltage v_L, or

$$12.4 - 12 = 0.4 = R_s i_L, \qquad R_s = \frac{0.4}{I_L} = 2.4\ \Omega$$

(b) Now that we have the Thevenin equivalent of the source, we connect the 8-Ω load across the Thevenin equivalent as shown in Fig. 4.62(b). Using voltage division, we obtain

$$v = \frac{8}{8 + 2.4}(12) = 9.231 \text{ V}$$

PRACTICE PROBLEM 4.16

The measured open-circuit voltage across a certain amplifier is 9 V. The voltage drops to 8 V when a 20-Ω loudspeaker is connected to the amplifier. Calculate the voltage when a 10-Ω loudspeaker is used instead.

Answer: 7.2 V.

4.10.2 Resistance Measurement

Although the ohmmeter method provides the simplest way to measure resistance, more accurate measurement may be obtained using the Wheatstone bridge. While ohmmeters are designed to measure resistance in low, mid, or high range, a Wheatstone bridge is used to measure resistance in the mid range, say, between 1 Ω and 1 MΩ. Very low values of resistances are measured with a *milliohmmeter*, while very high values are measured with a *Megger tester.*

Historical note: The bridge was invented by Charles Wheatstone (1802–1875), a British professor who also invented the telegraph, as Samuel Morse did independently in the United States.

The Wheatstone bridge (or resistance bridge) circuit is used in a number of applications. Here we will use it to measure an unknown resistance. The unknown resistance R_x is connected to the bridge as shown in Fig. 4.63. The variable resistance is adjusted until no current flows through the galvanometer, which is essentially a d'Arsonval movement operating as a sensitive current-indicating device like an ammeter in the microamp range. Under this condition $v_1 = v_2$, and the bridge is said to be *balanced*. Since no current flows through the galvanometer, R_1 and R_2 behave as though they were in series; so do R_3 and R_x. The fact that no current flows through the galvanometer also implies that $v_1 = v_2$. Applying the voltage division principle,

$$v_1 = \frac{R_2}{R_1 + R_2} v = v_2 = \frac{R_x}{R_3 + R_x} v \tag{4.29}$$

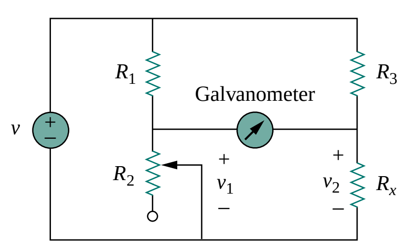


Figure 4.63 The Wheatstone bridge; R_x is the resistance to be measured.

Hence, no current flows through the galvanometer when

$$\frac{R_2}{R_1 + R_2} = \frac{R_x}{R_3 + R_x} \qquad \Longrightarrow \qquad R_2 R_3 = R_1 R_x$$

or

$$\boxed{R_x = \frac{R_3}{R_1} R_2} \tag{4.30}$$

If $R_1 = R_3$, and R_2 is adjusted until no current flows through the galvanometer, then $R_x = R_2$.

How do we find the current through the galvanometer when the Wheatstone bridge is *unbalanced*? We find the Thevenin equivalent (V_{Th} and R_{Th}) with respect to the galvanometer terminals. If R_m is the resistance of the galvanometer, the current through it under the unbalanced condition is

$$I = \frac{V_{\text{Th}}}{R_{\text{Th}} + R_m} \tag{4.31}$$

Example 4.18 will illustrate this.

EXAMPLE 4.17

In Fig. 4.63, $R_1 = 500\ \Omega$ and $R_3 = 200\ \Omega$. The bridge is balanced when R_2 is adjusted to be 125 Ω. Determine the unknown resistance R_x.

Solution:

Using Eq. (4.30),

$$R_x = \frac{R_3}{R_1} R_2 = \frac{200}{500} 125 = 50\ \ \Omega$$

PRACTICE PROBLEM 4.17

A Wheatstone bridge has $R_1 = R_3 = 1\ \text{k}\Omega$. R_2 is adjusted until no current flows through the galvanometer. At that point, $R_2 = 3.2\ \text{k}\Omega$. What is the value of the unknown resistance?

Answer: 3.2 kΩ.

EXAMPLE 4.18

The circuit in Fig. 4.64 represents an unbalanced bridge. If the galvanometer has a resistance of 40 Ω, find the current through the galvanometer.

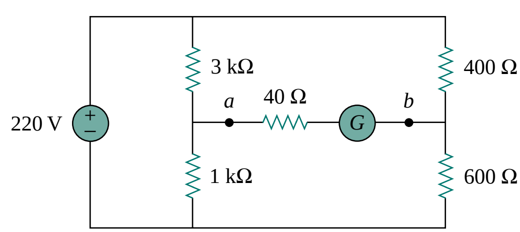


Figure 4.64 Unbalanced bridge of Example 4.18.

Solution:

We first need to replace the circuit by its Thevenin equivalent at terminals a and b. The Thevenin resistance is found using the circuit in Fig. 4.65(a). Notice that the 3-kΩ and 1-kΩ resistors are in parallel; so are the 400-Ω and 600-Ω resistors. The two parallel combinations form a series combination with respect to terminals a and b. Hence,

$$\begin{aligned} R_{\text{Th}} &= 3000 \parallel 1000 + 400 \parallel 600 \\ &= \frac{3000 \times 1000}{3000 + 1000} + \frac{400 \times 600}{400 + 600} = 750 + 240 = 990\ \Omega \end{aligned}$$

To find the Thevenin voltage, we consider the circuit in Fig. 4.65(b). Using the voltage division principle,

$$v_1 = \frac{1000}{1000 + 3000}(220) = 55\ \text{V}, \qquad v_2 = \frac{600}{600 + 400}(220) = 132\ \text{V}$$

Applying KVL around loop ab gives

$$-v_1 + V_{\text{Th}} + v_2 = 0 \qquad \text{or} \qquad V_{\text{Th}} = v_1 - v_2 = 55 - 132 = -77\ \text{V}$$

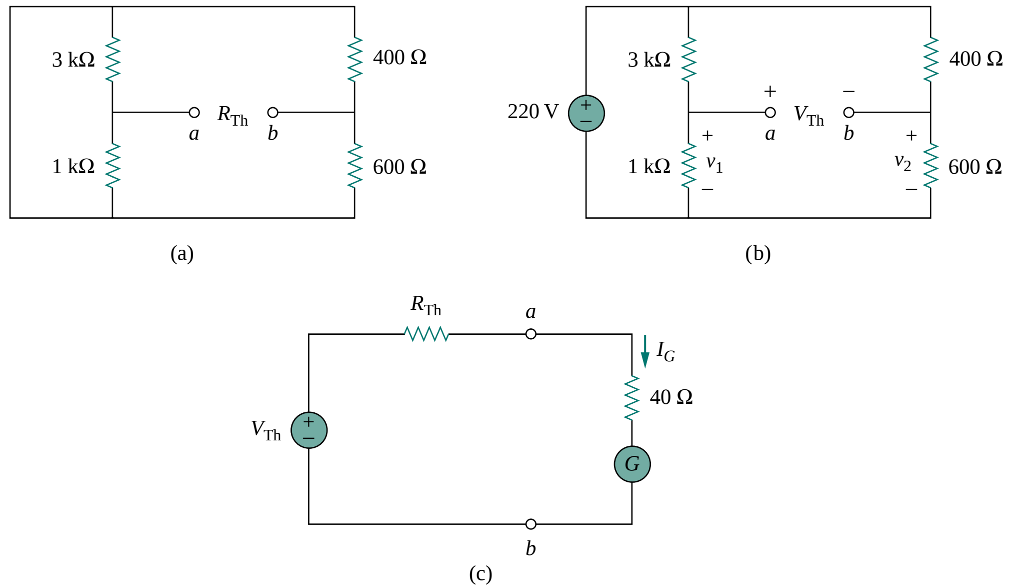


Figure 4.65 For Example 4.18: (a) Finding R_{Th}, (b) finding V_{Th}, (c) determining the current through the galvanometer.

Having determined the Thevenin equivalent, we find the current through the galvanometer using Fig. 4.65(c).

$$I_G = \frac{V_{Th}}{R_{Th} + R_m} = \frac{-77}{990 + 40} = -74.76 \text{ mA}$$

The negative sign indicates that the current flows in the direction opposite to the one assumed, that is, from terminal b to terminal a.

PRACTICE PROBLEM 4.18

Obtain the current through the galvanometer, having a resistance of 14 Ω, in the Wheatstone bridge shown in Fig. 4.66.

Answer: 64 mA.

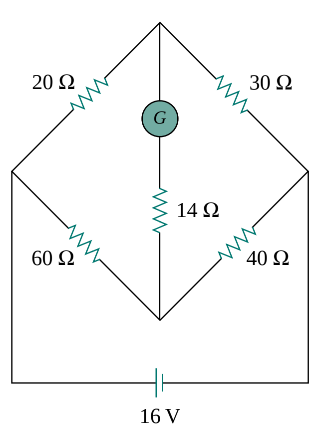


Figure 4.66 For Practice Prob. 4.18.

4.11 SUMMARY

1. A linear network consists of linear elements, linear dependent sources, and linear independent sources.
2. Network theorems are used to reduce a complex circuit to a simpler one, thereby making circuit analysis much simpler.
3. The superposition principle states that for a circuit having multiple independent sources, the voltage across (or current through) an element is equal to the algebraic sum of all the individual voltages (or currents) due to each independent source acting one at a time.
4. Source transformation is a procedure for transforming a voltage source in series with a resistor to a current source in parallel with a resistor, or vice versa.
5. Thevenin's and Norton's theorems allow us to isolate a portion of a network while the remaining portion of the network is replaced by an equivalent network. The Thevenin equivalent consists of a voltage source V_{Th} in series with a resistor R_{Th}, while the Norton equivalent consists of a current source I_N in parallel with a resistor R_N. The two theorems are related by source transformation.

$$R_N = R_{\text{Th}}, \qquad I_N = \frac{V_{\text{Th}}}{R_{\text{Th}}}$$

6. For a given Thevenin equivalent circuit, maximum power transfer occurs when $R_L = R_{\text{Th}}$, that is, when the load resistance is equal to the Thevenin resistance.
7. *PSpice* can be used to verify the circuit theorems covered in this chapter.
8. Source modeling and resistance measurement using the Wheatstone bridge provide applications for Thevenin's theorem.

REVIEW QUESTIONS

4.1 The current through a branch in a linear network is 2 A when the input source voltage is 10 V. If the voltage is reduced to 1 V and the polarity is reversed, the current through the branch is:

(a) −2 (b) −0.2 (c) 0.2
(d) 2 (e) 20

4.2 For superposition, it is not required that only one independent source be considered at a time; any number of independent sources may be considered simultaneously.

(a) True (b) False

4.3 The superposition principle applies to power calculation.

(a) True (b) False

4.4 Refer to Fig. 4.67. The Thevenin resistance at terminals a and b is:

(a) 25 Ω (b) 20 Ω
(c) 5 Ω (d) 4 Ω

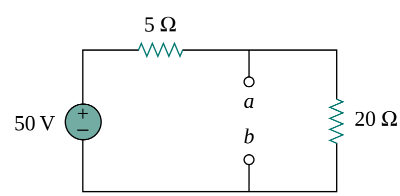


Figure 4.67 For Review Questions 4.4 to 4.6.

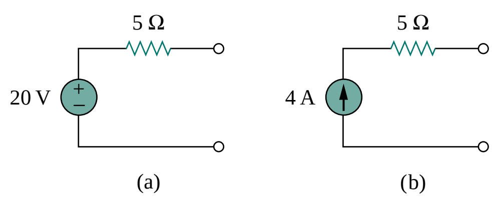
4.5 The Thevenin voltage across terminals a and b of the circuit in Fig. 4.67 is:

(a) 50 V (b) 40 V
(c) 20 V (d) 10 V

4.6 The Norton current at terminals a and b of the circuit in Fig. 4.67 is:
(a) 10 A (b) 2.5 A
(c) 2 A (d) 0 A

4.7 The Norton resistance R_N is exactly equal to the Thevenin resistance R_{Th}.
(a) True (b) False

4.8 Which pair of circuits in Fig. 4.68 are equivalent?
(a) a and b (b) b and d
(c) a and c (d) c and d



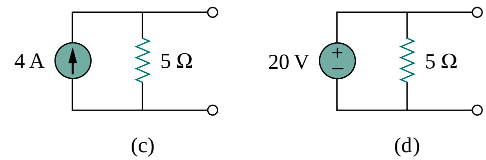


Figure 4.68 For Review Question 4.8.

4.9 A load is connected to a network. At the terminals to which the load is connected, $R_{Th} = 10\ \Omega$ and $V_{Th} = 40$ V. The maximum power supplied to the load is:
(a) 160 W (b) 80 W
(c) 40 W (d) 1 W

4.10 The source is supplying the maximum power to the load when the load resistance equals the source resistance.
(a) True (b) False

Answers: 4.1b, 4.2a, 4.3b, 4.4d, 4.5b, 4.6a, 4.7a, 4.8c, 4.9c, 4.10b.

PROBLEMS

Section 4.2 Linearity Property

4.1 Calculate the current i_o in the current of Fig. 4.69. What does this current become when the input voltage is raised to 10 V?

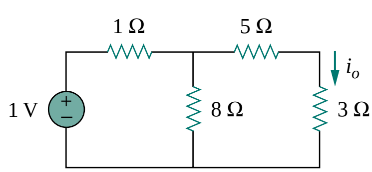


Figure 4.69 For Prob. 4.1.

4.2 Find v_o in the circuit of Fig. 4.70. If the source current is reduced to 1 μA, what is v_o?

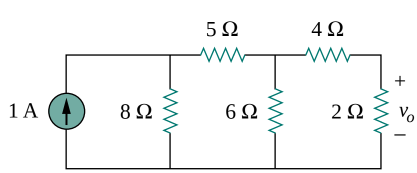


Figure 4.70 For Prob. 4.2.

4.3 (a) In the circuit in Fig. 4.71, calculate v_o and i_o when $v_s = 1$ V.
(b) Find v_o and i_o when $v_s = 10$ V.
(c) What are v_o and i_o when each of the 1-Ω resistors is replaced by a 10-Ω resistor and $v_s = 10$ V?

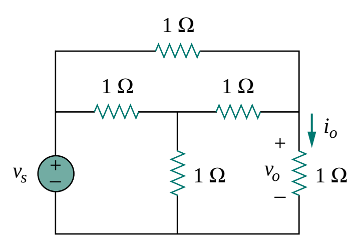


Figure 4.71 For Prob. 4.3.

4.4 Use linearity to determine i_o in the circuit of Fig. 4.72.

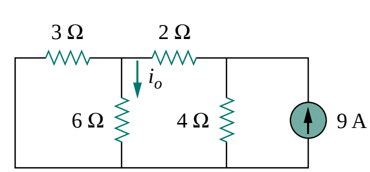


Figure 4.72 For Prob. 4.4.

4.5 For the circuit in Fig. 4.73, assume $v_o = 1$ V, and use linearity to find the actual value of v_o.

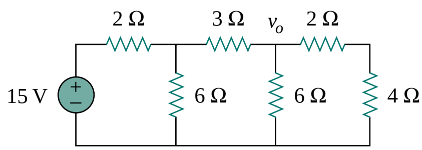


Figure 4.73 For Prob. 4.5.

Section 4.3 Superposition

4.6 Apply superposition to find i in the circuit of Fig. 4.74.

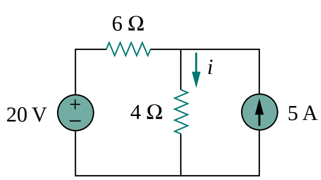


Figure 4.74 For Prob. 4.6.

4.7 Given the circuit in Fig. 4.75, calculate i_x and the power dissipated by the 10-Ω resistor using superposition.

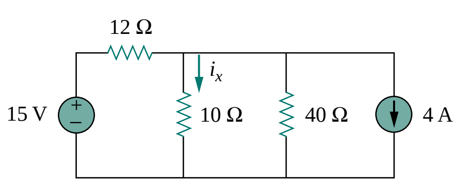


Figure 4.75 For Prob. 4.7.

4.8 For the circuit in Fig. 4.76, find the terminal voltage V_{ab} using superposition.

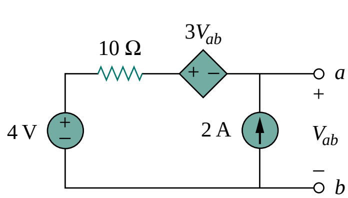


Figure 4.76 For Prob. 4.8.

4.9 Use superposition principle to find i in Fig. 4.77.

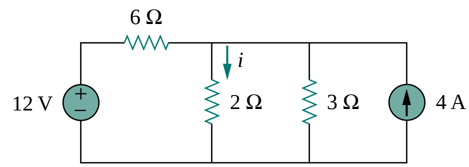


Figure 4.77 For Prob. 4.9.

4.10 Determine v_o in the circuit of Fig. 4.78 using the superposition principle.

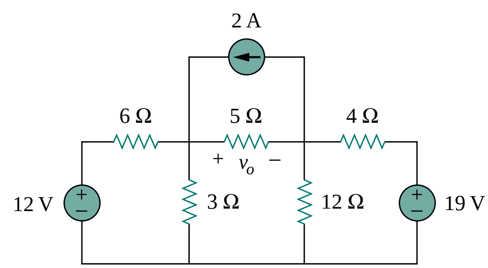


Figure 4.78 For Prob. 4.10.

4.11 Apply the superposition principle to find v_o in the circuit of Fig. 4.79.

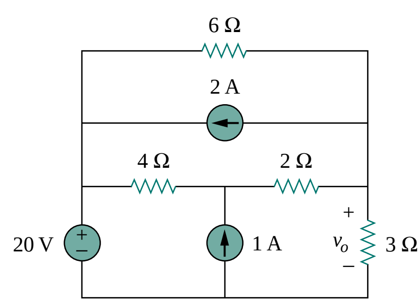


Figure 4.79 For Prob. 4.11.

4.12 For the circuit in Fig. 4.80, use superposition to find i. Calculate the power delivered to the 3-Ω resistor.

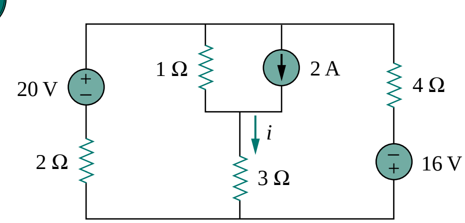


Figure 4.80 For Probs. 4.12 and 4.45.

4.13 Given the circuit in Fig. 4.81, use superposition to get i_o.

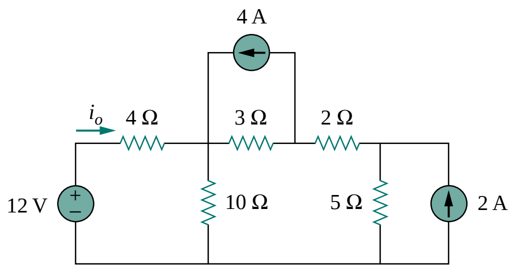


Figure 4.81 For Probs. 4.13 and 4.23.

4.14 Use superposition to obtain v_x in the circuit of Fig. 4.82. Check your result using *PSpice*.

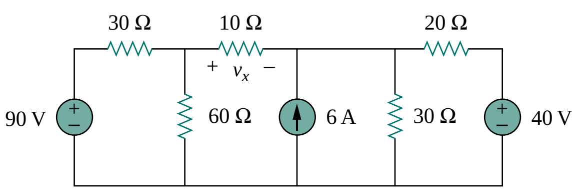


Figure 4.82 For Prob. 4.14.

4.15 Find v_x in Fig. 4.83 by superposition.

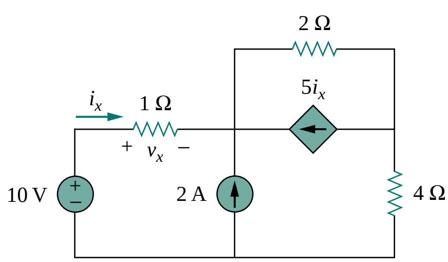


Figure 4.83 For Prob. 4.15.

4.16 Use superposition to solve for i_x in the circuit of Fig. 4.84.

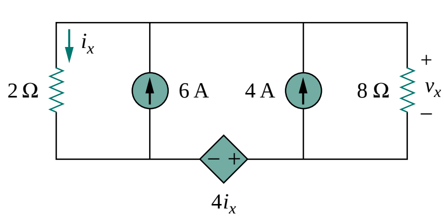


Figure 4.84 For Prob. 4.16.

Section 4.4 Source Transformation

4.17 Find i in Prob. 4.9 using source transformation.

4.18 Apply source transformation to determine v_o and i_o in the circuit in Fig. 4.85.

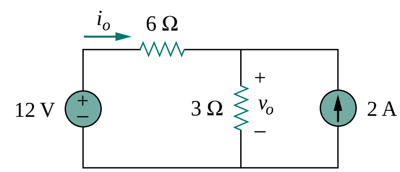


Figure 4.85 For Prob. 4.18.

4.19 For the circuit in Fig. 4.86, use source transformation to find i.

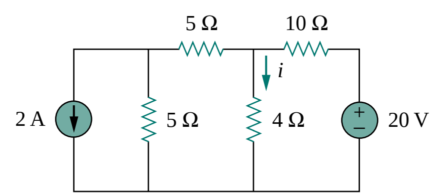


Figure 4.86 For Prob. 4.19.

4.20 Obtain v_o in the circuit of Fig. 4.87 using source transformation. Check your result using *PSpice*.

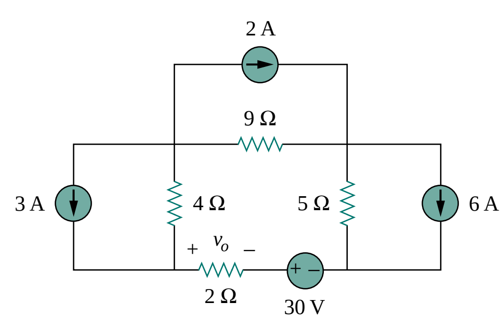


Figure 4.87 For Prob. 4.20.

4.21 Use source transformation to solve Prob. 4.14.

4.22 Apply source transformation to find v_x in the circuit of Fig. 4.88.

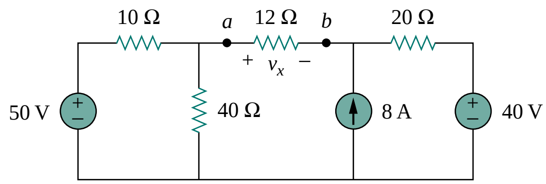


Figure 4.88 For Probs. 4.22 and 4.32.

4.23 Given the circuit in Fig. 4.81, use source transformation to find i_o.

4.24 Use source transformation to find v_o in the circuit of Fig. 4.89.

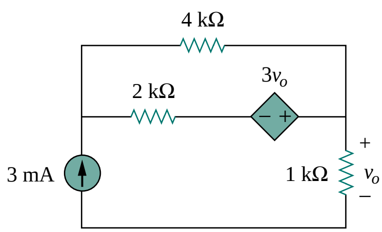


Figure 4.89 For Prob. 4.24.

4.25 Determine v_x in the circuit of Fig. 4.90 using source transformation.

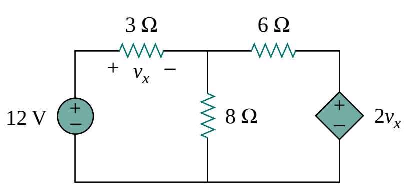


Figure 4.90 For Prob. 4.25.

4.26 Use source transformation to find i_x in the circuit of Fig. 4.91.

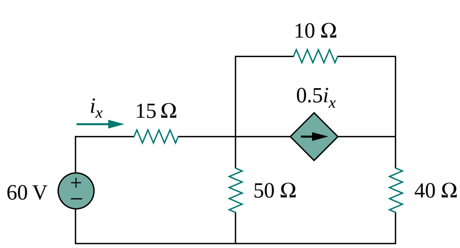


Figure 4.91 For Prob. 4.26.

Sections 4.5 and 4.6 Thevenin's and Norton's Theorems

4.27 Determine R_{Th} and V_{Th} at terminals 1-2 of each of the circuits in Fig. 4.92.

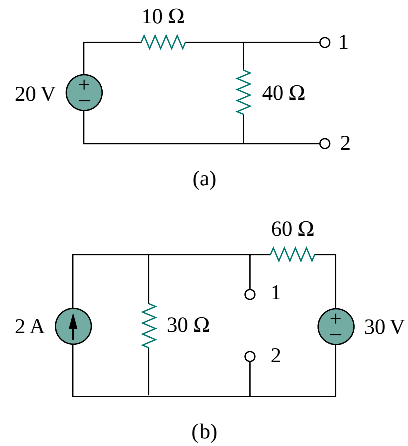


Figure 4.92 For Probs. 4.27 and 4.37.

4.28 Find the Thevenin equivalent at terminals *a*-*b* of the circuit in Fig. 4.93.

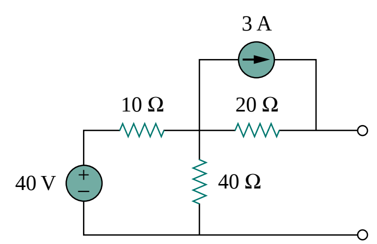


Figure 4.93 For Probs. 4.28 and 4.39.

4.29 Use Thevenin's theorem to find v_o in Prob. 4.10.

4.30 Solve for the current i in the circuit of Fig. 4.94 using Thevenin's theorem. (*Hint:* Find the Thevenin equivalent across the 12-Ω resistor.)

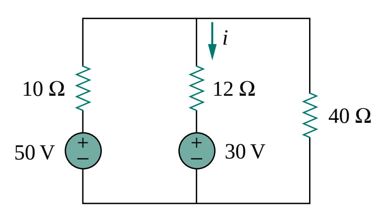


Figure 4.94 For Prob. 4.30.

4.31 For Prob. 4.8, obtain the Thevenin equivalent at terminals *a*-*b*.

4.32 Given the circuit in Fig. 4.88, obtain the Thevenin equivalent at terminals a-b and use the result to get v_x.

*__4.33__ For the circuit in Fig. 4.95, find the Thevenin equivalent between terminals a and b.

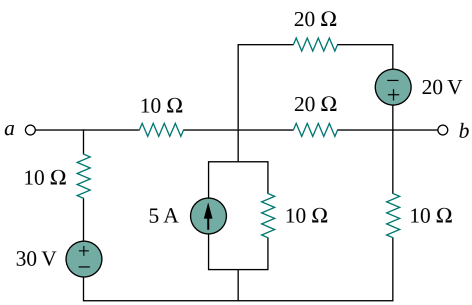


Figure 4.95 For Prob. 4.33.

4.34 Find the Thevenin equivalent looking into terminals a-b of the circuit in Fig. 4.96 and solve for i_x.

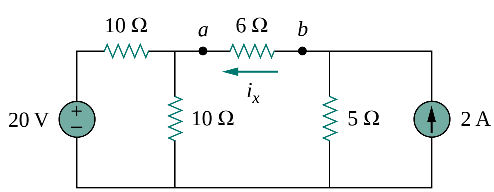


Figure 4.96 For Prob. 4.34.

4.35 For the circuit in Fig. 4.97, obtain the Thevenin equivalent as seen from terminals:
(a) a-b (b) b-c

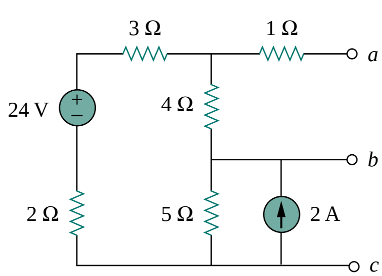


Figure 4.97 For Prob. 4.35.

4.36 Find the Norton equivalent of the circuit in Fig. 4.98.

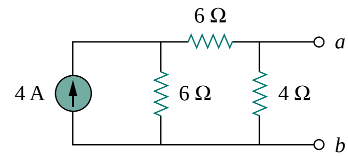


Figure 4.98 For Prob. 4.36.

4.37 Obtain R_N and I_N at terminals 1 and 2 of each of the circuits in Fig. 4.92.

4.38 Determine the Norton equivalent at terminals a-b for the circuit in Fig. 4.99.

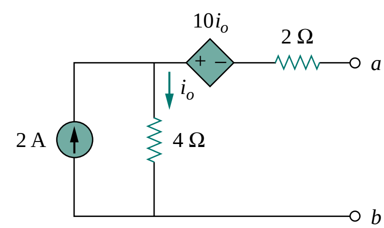


Figure 4.99 For Prob. 4.38.

4.39 Find the Norton equivalent looking into terminals a-b of the circuit in Fig. 4.93.

4.40 Obtain the Norton equivalent of the circuit in Fig. 4.100 to the left of terminals a-b. Use the result to find current i.

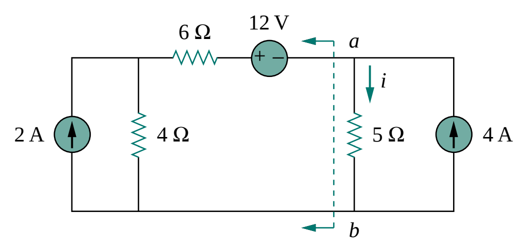


Figure 4.100 For Prob. 4.40.

4.41 Given the circuit in Fig. 4.101, obtain the Norton equivalent as viewed from terminals:
(a) a-b (b) c-d

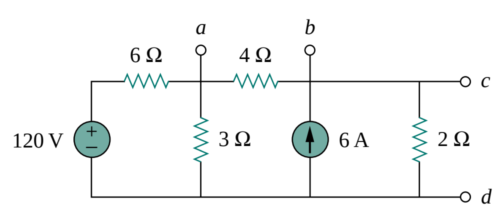


Figure 4.101 For Prob. 4.41.

*An asterisk indicates a challenging problem.

4.42 For the transistor model in Fig. 4.102, obtain the Thevenin equivalent at terminals *a-b*.

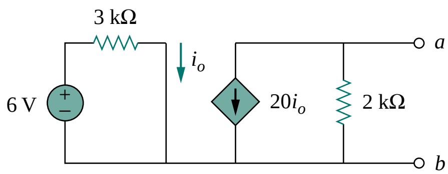


Figure 4.102 For Prob. 4.42.

4.43 Find the Norton equivalent at terminals *a-b* of the circuit in Fig. 4.103.

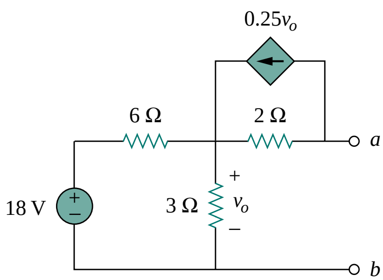


Figure 4.103 For Prob. 4.43.

*__4.44__ Obtain the Norton equivalent at terminals *a-b* of the circuit in Fig. 4.104.

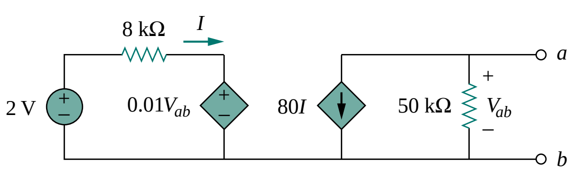


Figure 4.104 For Prob. 4.44.

4.45 Use Norton's theorem to find current i in the circuit of Fig. 4.80.

4.46 Obtain the Thevenin and Norton equivalent circuits at the terminals *a-b* for the circuit in Fig. 4.105.

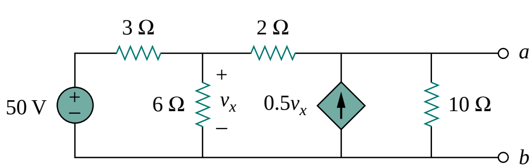


Figure 4.105 For Probs. 4.46 and 4.65.

4.47 The network in Fig. 4.106 models a bipolar transistor common-emitter amplifier connected to a load. Find the Thevenin resistance seen by the load.

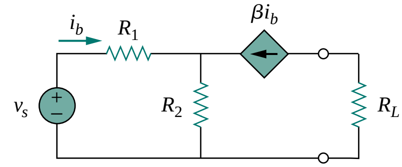


Figure 4.106 For Prob. 4.47.

4.48 Determine the Thevenin and Norton equivalents at terminals *a-b* of the circuit in Fig. 4.107.

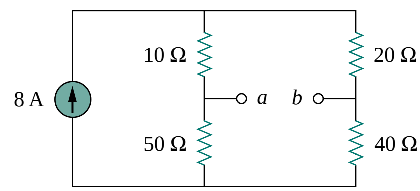


Figure 4.107 For Probs. 4.48 and 4.66.

*__4.49__ For the circuit in Fig. 4.108, find the Thevenin and Norton equivalent circuits at terminals *a-b*.

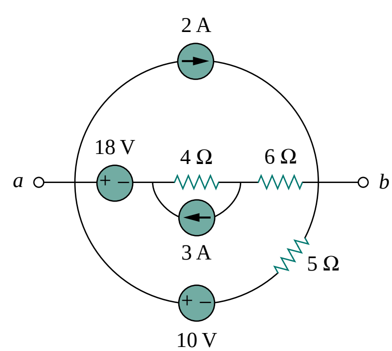


Figure 4.108 For Probs. 4.49 and 4.67.

*__4.50__ Obtain the Thevenin and Norton equivalent circuits at terminals *a-b* of the circuit in Fig. 4.109.

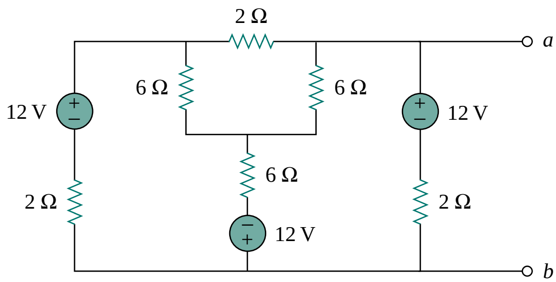


Figure 4.109 For Prob. 4.50.

*4.51 Find the Thevenin equivalent of the circuit in Fig. 4.110.

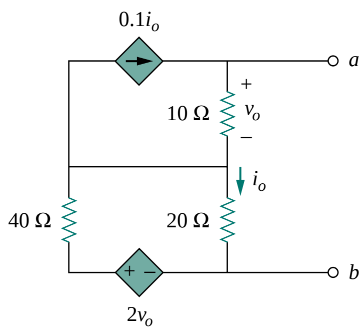


Figure 4.110 For Prob. 4.51.

4.52 Find the Norton equivalent for the circuit in Fig. 4.111.

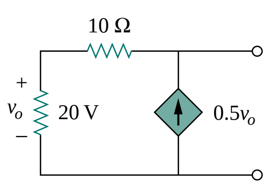


Figure 4.111 For Prob. 4.52.

4.53 Obtain the Thevenin equivalent seen at terminals *a*-*b* of the circuit in Fig. 4.112.

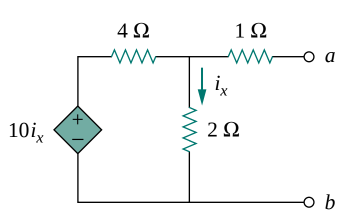


Figure 4.112 For Prob. 4.53.

Section 4.8 Maximum Power Transfer

4.54 Find the maximum power that can be delivered to the resistor R in the circuit in Fig. 4.113.

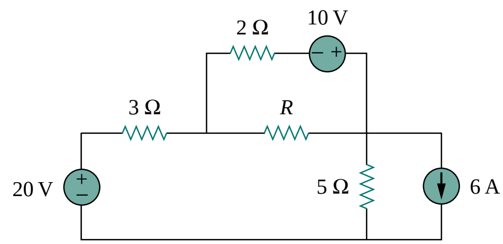


Figure 4.113 For Prob. 4.54.

4.55 Refer to Fig. 4.114. For what value of R is the power dissipated in R maximum? Calculate that power.

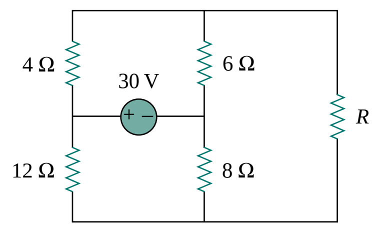


Figure 4.114 For Prob. 4.55.

*4.56 Compute the value of R that results in maximum power transfer to the 10-Ω resistor in Fig. 4.115. Find the maximum power.

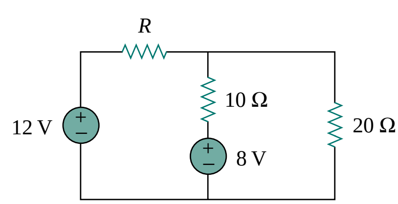


Figure 4.115 For Prob. 4.56.

4.57 Find the maximum power transferred to resistor R in the circuit of Fig. 4.116.

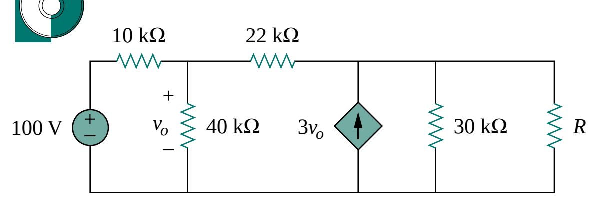


Figure 4.116 For Prob. 4.57.

4.58 For the circuit in Fig. 4.117, what resistor connected across terminals a-b will absorb maximum power from the circuit? What is that power?

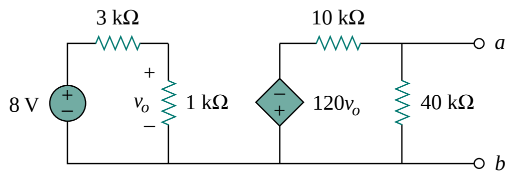


Figure 4.117 For Prob. 4.58.

4.59 (a) For the circuit in Fig. 4.118, obtain the Thevenin equivalent at terminals a-b.
(b) Calculate the current in $R_L = 8\ \Omega$.
(c) Find R_L for maximum power deliverable to R_L.
(d) Determine that maximum power.

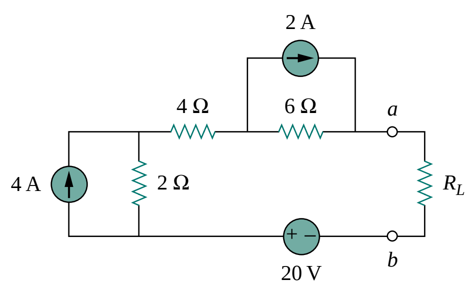


Figure 4.118 For Prob. 4.59.

4.60 For the bridge circuit shown in Fig. 4.119, find the load R_L for maximum power transfer and the maximum power absorbed by the load.

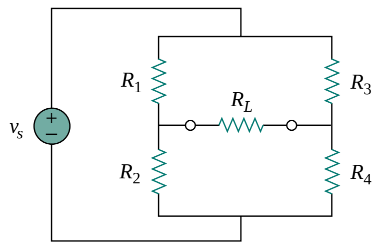


Figure 4.119 For Prob. 4.60.

4.61 For the circuit in Fig. 4.120, determine the value of R such that the maximum power delivered to the load is 3 mW.

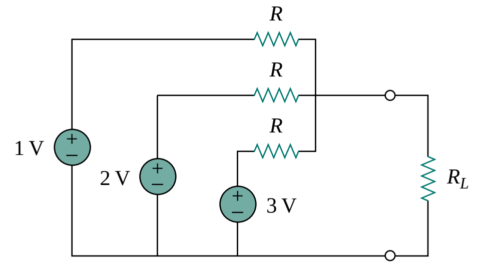


Figure 4.120 For Prob. 4.61.

Section 4.9 Verifying Circuit Theorems with *PSpice*

4.62 Solve Prob. 4.28 using *PSpice*.

4.63 Use *PSpice* to solve Prob. 4.35.

4.64 Use *PSpice* to solve Prob. 4.42.

4.65 Obtain the Thevenin equivalent of the circuit in Fig. 4.105 using *PSpice*.

4.66 Use *PSpice* to find the Thevenin equivalent circuit at terminals a-b of the circuit in Fig. 4.107.

4.67 For the circuit in Fig. 4.108, use *PSpice* to find the Thevenin equivalent at terminals a-b.

Section 4.10 Applications

4.68 A battery has a short-circuit current of 20 A and an open-circuit voltage of 12 V. If the battery is connected to an electric bulb of resistance 2 Ω, calculate the power dissipated by the bulb.

4.69 The following results were obtained from measurements taken between the two terminals of a resistive network.

Terminal Voltage	12 V	0 V
Terminal Current	0 V	1.5 A

Find the Thevenin equivalent of the network.

4.70 A black box with a circuit in it is connected to a variable resistor. An ideal ammeter (with zero resistance) and an ideal voltmeter (with infinite resistance) are used to measure current and voltage as shown in Fig. 4.121. The results are shown in the table below.

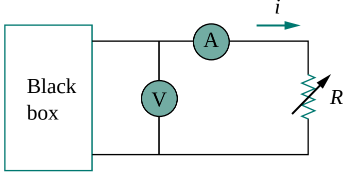


Figure 4.121 For Prob. 4.70.

(a) Find i when $R = 4\ \Omega$.

(b) Determine the maximum power from the box.

$R(\Omega)$	V(V)	i(A)
2	3	1.5
8	8	1.0
14	10.5	0.75

4.71 A transducer is modeled with a current source I_s and a parallel resistance R_s. The current at the terminals of the source is measured to be 9.975 mA when an ammeter with an internal resistance of 20 Ω is used.

(a) If adding a 2-kΩ resistor across the source terminals causes the ammeter reading to fall to 9.876 mA, calculate I_s and R_s.

(b) What will the ammeter reading be if the resistance between the source terminals is changed to 4 kΩ?

4.72 The Wheatstone bridge circuit shown in Fig. 4.122 is used to measure the resistance of a strain gauge. The adjustable resistor has a linear taper with a maximum value of 100 Ω. If the resistance of the strain gauge is found to be 42.6 Ω, what fraction of the full slider travel is the slider when the bridge is balanced?

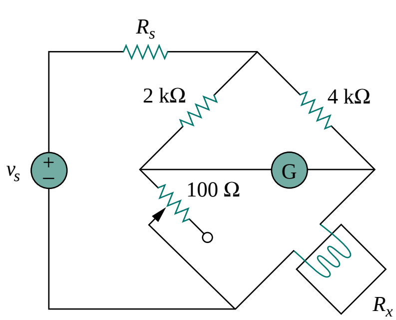


Figure 4.122 For Prob. 4.72.

4.73 (a) In the Wheatstone bridge circuit of Fig. 4.123, select the values of R_1 and R_3 such that the bridge can measure R_x in the range of 0–10 Ω.

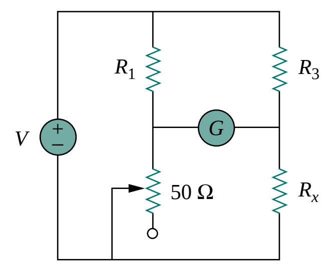


Figure 4.123 For Prob. 4.73.

(b) Repeat for the range of 0–100 Ω.

***4.74** Consider the bridge circuit of Fig. 4.124. Is the bridge balanced? If the 10-kΩ resistor is replaced by an 18-kΩ resistor, what resistor connected between terminals a-b absorbs the maximum power? What is this power?

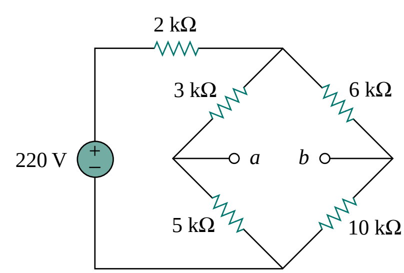


Figure 4.124 For Prob. 4.74.

COMPREHENSIVE PROBLEMS

4.75 The circuit in Fig. 4.125 models a common-emitter transistor amplifier. Find i_x using source transformation.

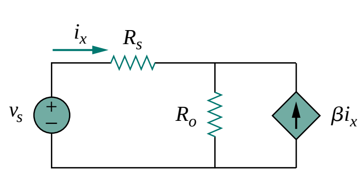


Figure 4.125 For Prob. 4.75.

4.76 An attenuator is an interface circuit that reduces the voltage level without changing the output resistance.

(a) By specifying R_s and R_p of the interface circuit in Fig. 4.126, design an attenuator that will meet the following requirements:

$$\frac{V_o}{V_g} = 0.125, \qquad R_{eq} = R_{Th} = R_g = 100\ \Omega$$

(b) Using the interface designed in part (a), calculate the current through a load of $R_L = 50\ \Omega$ when $V_g = 12$ V.

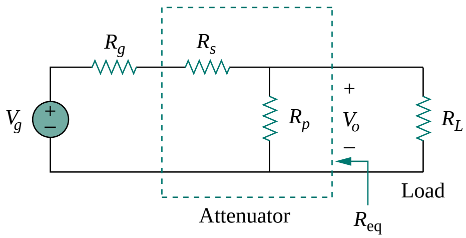


Figure 4.126 For Prob. 4.76.

*4.77 A dc voltmeter with a sensitivity of 20 kΩ/V is used to find the Thevenin equivalent of a linear network. Readings on two scales are as follows:

(a) 0–10 V scale: 4 V (b) 0–50 V scale: 5 V

Obtain the Thevenin voltage and the Thevenin resistance of the network.

*4.78 A resistance array is connected to a load resistor R and a 9-V battery as shown in Fig. 4.127.

(a) Find the value of R such that $V_o = 1.8$ V.

(b) Calculate the value of R that will draw the maximum current. What is the maximum current?

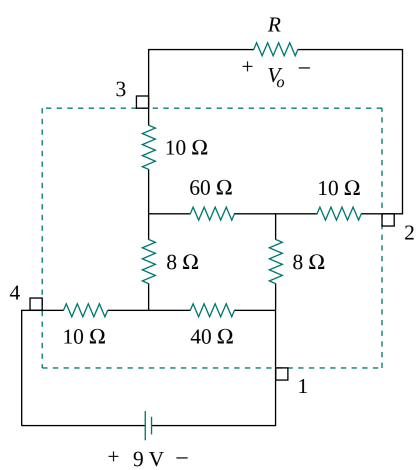


Figure 4.127 For Prob. 4.78.

4.79 A common-emitter amplifier circuit is shown in Fig. 4.128. Obtain the Thevenin equivalent to the left of points B and E.

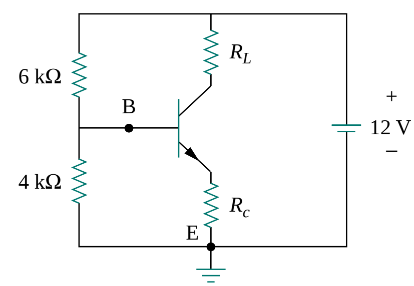


Figure 4.128 For Prob. 4.79.

*4.80 For Practice Prob. 4.17, determine the current through the 40-Ω resistor and the power dissipated by the resistor.

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